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Two Weak Points of the Enhanced Indispensability Argument – Domain of the Argument and Definition of Indispensability

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ABSTRACT: The contemporary Platonists in the philosophy of mathematics argue that mathematical objects exist. One of the arguments by which they support this standpoint is the so-called Enhanced Indispensability Argument (EIA). This paper aims at pointing out the difficulties inherent to the EIA. The first is contained in the vague formulation of the Argument, which is the reason why not even an approximate scope of the set objects whose existence is stated by the Argument can be established. The second problem is reflected in the vagueness of the very term indispensability, which is essential to the Argument. The paper will remind of a recent definition of the concept of indispensability of a mathematical object, reveal its deficiency and propose an improvement of this definition. Following this, we will deal with one of the consequences of the arbitrary employment of the concept of indispensability of a mathematical theory. We will propose a definition of this concept as well, in accordance with the common intuition about it. Eventually, on the basis of these two definitions, the paper will describe the relation between these two concepts, in the attempt to clarify the conceptual apparatus of the EIA.

KEYWORDS: Platonism – Enhanced Indispensability Argument – definition of indispensability – intuition.

1. Introduction

The contemporary Platonists in the philosophy of mathematics maintain that mathematical objects have an existence. However, they do not seem to be able to provide a more detailed explanation of the nature and features of that existence. To sustain their attitude, they use various arguments. One of these is the so-called Enhanced Indispensability Argument, formulated explicitly several years ago by Alan Baker, who used the following modal syllogism (cf. Baker 2009, 613):

(1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.
(2) Mathematical objects play an indispensable explanatory role in science.
(3) Hence, we ought rationally to believe in the existence of mathematical objects.

It could be said that Baker's formulation is an explicit consequence of the long-term discussion held on the relation Nominalism-Platonism[1] regarding the necessity to specify the *sort* of indispensability which mathematics could treat as a scientific subject.[2] The idea behind the Argument is quite natural. Broadly speaking, if science describes and *explains* phenomena and objects which doubtlessly exist, then such a feature – an existence – must also be attributed to the tools used in those explanations. Since, among other reasons, we use mathematical objects to explain empirical phenomena, we can conclude that those objects do exist. Historically speaking, the Enhanced Indispensability Argument (henceforth EIA) is an "improved" version of the so-called Quine-Putnam indispensability argument (IA), according to which the role that mathematical objects have in describing and explaining empirical phenomena is reduced to quantification and indexing of the physical objects.[3] In addition

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1 See, for example, Melia (2002) for the Nominalist, and Colyvan (2002) for the Platonist side.

2 By the word 'science' in this text, we will imply empirical sciences, such as physics, chemistry, biology, etc.

3 The classic position in the reference books is occupied by Putnam (1971, 65). See, for example, Melia (2000, 455), Yablo (2000, 197), Colyvan (2001, 10). Nevertheless, there are authors who have been trying to prove that neither Putnam, nor Quine can be

to this, the EIA places an emphasis on the indispensability of the explanatory role of the mathematical objects in the empirical science.

The aim of this paper is to draw attention to the difficulties entailed in the EIA. The first difficulty is reflected in the vague formulation of the EIA. This vagueness is the reason why it is not possible to determine the scope of the set of objects whose existence is stated by the Argument.[4] The lack of precision, as this paper will show, even though prevalently technical in nature, reminds of that precarious and vital question which has remained unanswered since the beginnings of Platonism.[5] The other difficulty is reflected in yet another imprecision. Namely, it refers to the notion of the *indispensable explanatory role*,[6] the meaning of which had not been specified until recently, which could have resulted in different interpretations of the concept and, consequently, in different interpretations of the EIA. For this reason, the major attention will be given to the concept of (in)dispensability. More precisely, a recent proposal for the definition of the *indispensability of a mathematical object* will be recalled here; its drawback will be pointed out and a possible improvement of this definition will be suggested. Following this, the paper will deal with an unpleasant consequence of the arbitrary use of the concept *indispensability of a mathematical theory*. It is evident in the intuitively hardly graspable relationship between indispensability of an object and indispensability of a theory. We will, therefore, propose a definition of the indispensability of a mathematical theory trying to follow the line of the intuition generally held about this notion. Finally, on the basis of the two definitions – one improved and the other only suggested – we will describe the relation between these two concepts thus attempting to clarify, at least to some extent, the conceptual apparatus used in the EIA.

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accredited with the main part of the indispensability argument. For more information, see Liggins (2008).

4 There are opinions that scope does not matter in the case of the IA, but that what matters is a question of its specificity (cf. Baker 2003, 52). It rather seems that in the case of the EIA, as a more explicit and more precise argument, the question of scope cannot be declared as a peripheral one.

5 Is it possible to speak of the existence of *only some* mathematical objects?

6 For the reasons of brevity and clarity, in most cases henceforth the simple term *indispensability* of a mathematical object or mathematical theory will be used instead of *indispensable explanatory role* of a mathematical object or mathematical theory.

2. Baker's example – dilemmas

When it comes to the explanatory role of mathematics in science, there are several analyses directly related to the EIA. Some of the authors point to the impossibility of reaching any conclusion about the existence of mathematical objects on the basis of their explanatory role in science (see Bangu 2008), whereas the others claim that mathematical objects possess no explanatory capacity whatsoever in the case of empirical events (cf. Daly and Langford 2009). Also, there are authors who adhere to the standpoint that mathematical objects and models do not explain empirical phenomena in a genuine way, but only represent them in one of the possible ways (cf. Saatsi 2011), while others observe that the expression "indispensable explanatory role" has been used imprecisely in the EIA (see Molini 2014). The latter observation will be the main focus of this discussion. Let us be reminded of the famous cicada example, the common point, used to illustrate the mathematical explanation of an empirical phenomenon:

> The example featured the life cycle of the periodical cicada, an insect whose two North American subspecies spend 13 years and 17 years, respectively, underground in larval form before emerging briefly as adults. One question raised by biologists is: why are these life cycles prime? It turns out that a couple of explanations have been given that rely on certain number theoretic results to show that prime cycles minimize overlap with other periodical organisms. Avoiding overlap is beneficial whether the other organisms are predators, or whether they are different subspecies… (Baker 2009, 614)

> For example, a prey with a 12-year cycle will meet – every time it appears – properly synchronized predators appearing every 1, 2, 3, 4, 6 or 12 years, whereas a mutant with a 13-year period has the advantage of being subject to fewer predators. (Goles et al. 2001, 33)

This seems to be an example of a purely physical phenomenon being explained by mathematical tools. It applies one of the basic facts of the number theory. Since the prime number can only be divided by itself and by 1, the cicada whose life span equals a prime number has more chance of survival than the cicada whose life span equals a composite number, because the latter encounters a larger number of predators during life cycles than the former. However, it is not clear that the above example is the case where mathematical objects play an *indispensable* explanatory role. Baker does not find it necessary to define the notion of indispensability, presumably considering its meaning as intuitively sufficiently clear. As we can see, the domain of the attribute *indispensable* is considerably broad. Mathematics can play an indispensable explanatory role in science, and so does a mathematical apparatus or a mathematical object (see Baker 2009, 613-614). The indispensability of the mathematical object *O* for the explanation of the physical phenomenon *P* is nonformally understood as the impossibility to explain the phenomenon *P* without the use of the object *O* and its accompanying features. Therefore, in this case, to explain the phenomenon *P*, no other mathematical object can be helpful. Moreover, no object whatsoever can be used for the purposes of explanation.

Before turning to the analysis of the concept of indispensability, let us accept it intuitively, as Baker did, and let us return briefly to the EIA. In the formulation of the Argument many imprecisions can be found, which could create additional confusion. What, exactly, is it about? The conclusion of the EIA tells us that we ought to rationally believe in the existence of mathematical objects. It is a rather vague formulation of a potentially very important proposition, which can create various interpretations of the EIA. Namely, it is not clear whether we ought to believe or not in the existence of *all* or just *some* of the mathematical objects. This question may seem not so important at first; however, the answer to it fundamentally determines not only the further stages in clarification of the indispensability concept and defense of the EIA, but also the consistency of the Platonist attitude on the existence of mathematical objects (see Baker 2003, 53). To answer this question, it is necessary to solve the corresponding detail in the second premise of the EIA first. In other words, we should establish whether *all* or just *some* of the mathematical objects play an indispensable explanatory role in science. It is as if Baker, as well as those who used the formulation for the purpose of analyses and criticism (see, e.g. Molinini 2014), has not omitted the potential quantifier by coincidence, leaving thus a room for the possibility of various interpretations on the one hand, perhaps for the improvements as well, and rendering all criticism easier on the other. The imprecision in the definition of the EIA, however, with its lack of quantifier, cannot be a support to Platonism.[7]

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[7] If we were to express nominalist point of view with an opinion that there do not exist any abstract (mathematical) objects (see Baker 2003, 49), then the question about the EIA domain could easily be circumvented with the following answer: the primary

We cannot know with certainty what idea Baker had in mind when he formulated the EIA. Nevertheless, analyzing his famous cicada example, it may be deduced that he gravitates more towards the particular quantifier premise:

(2a) *Some* mathematical objects play an indispensable explanatory role in science;

And, consequently, towards the conclusion:

(3a) Hence, we ought rationally to believe in the existence of *some* mathematical objects.

Namely, by means of the cicada example, he illustrates the proposition about the existence of mathematical explanation in science, as well as the indispensability of mathematical apparatus, which suffices as the proof of a proposition such as *some As are Bs*. One single example, without any attempt to systematically find a role of every mathematical object used in the explanation of physical phenomena is, needless to say, far from endeavors to explain that all mathematical objects are indispensable for explaining physical phenomena. If such is the case, then we could speak about the existence of a mathematical object, more precisely, those mathematical objects that are indispensable for the explanation of physical phenomena. On the other hand, we would allow that other mathematical objects, about whose existence we do know, do not exist, and, also, that some of them do exist without our knowledge of them at the moment, since we perhaps do not know yet about a mathematical explanation of the physical phenomenon in which those mathematical objects are used. If we are to follow this line of argument, let us consider a mode to use the EIA in the cicada example. We could, for example, claim that numbers 13 and 17 exist or, to extend it, that all prime numbers exist, even though we could have given the explanation in this case without using the concept of the prime number, but using only the feature of divisibility common to all prime numbers, 13 and 17 included. If we interpret the EIA in a more flexible way, we could claim

purpose of the EIA is to refute nominalism. The existence of just one abstract mathematical object is enough to do this, hence there is a sense in which the EIA can succeed without addressing the scope. However, could we possibly accept that the main mission of the EIA is rebuttal of nominalism, without an attempt to create a systematic tool supportive of Platonism? We do not encounter a support for such a viewpoint in Baker's recent texts (cf. Baker 2005; 2009; 2015).

that there are composite numbers as well, because without comparing them with 13 and 17, we would not be able to understand the “advantages” of the prime numbers in this particular example in the first place. Nonetheless, no matter how flexibly we understand the application of the EIA, on the basis of this physical phenomenon and the EIA, we will not be able to claim the existence of some other objects of the Number Theory for certain, such as, for example, Euler’s function,[8] and, in particular, those objects which do not belong do the Number Theory, such as Polish space,[9] an object of the general topology. Indeed, as the mentioned objects are not used in the specific example, and as it has not been clearly indicated that they would ever be used for explaining a physical phenomenon, we cannot speak of their existence on the basis of the EIA. We can, therefore, speak of two levels of mathematical objects: of the “privileged” ones, which exist, and of those which do not have such a status, at least not at present. Evidently, the idea to use the EIA in order to prove the existence of *just some* of the mathematical objects appears rather unsustainable and easily discardable. Similar to this, the “partial” Platonism, seen recently in the philosophy of mathematics, was short winged as well.[10]

Let us return to the concept of indispensability. Baker regarded it as intuitively clear, although he must have been well aware that the majority of objections to the EIA were to be expected on that very point. Namely, from the mathematical as well as layman’s standpoint, the question highly expected is: in which way do we choose the mathematical apparatus for explaining a physical phenomenon? Is this choice an unambiguous process and what directs it? Is the whole process of selection an arbitrary one, carried out within random

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8 Euler’s function φ maps an arbitrary natural number *m* into the number of integers from 0 to $m - 1$ that are relatively prime to *m*. For example, $\varphi(1) = \varphi(2) = 1$, $\varphi(3) = \varphi(4) = 2$, $\varphi(5) = 4$. See Erdos and Suranyi (2003, 58).

9 A Polish space is a separable and completely metrizable topological space. There are two fundamental examples of Polish spaces. The first one is the Baire space N^N consisting of all sequences of natural numbers. The second one is the Cantor space 2^N consisting of all sequences of 0’s and 1’s. See Dodos (2010).

10 In Maddy (1990) we find an extremely odd idea about existence of only those mathematical objects which have a practical application, whereas other objects’ existence is denied. The author abandoned that position afterwards, as it can be seen in Maddy (1997).

circumstances, such as the affinity of the researcher, the current state of development of one of the mathematical theories, practical interests, etc.? Baker stated three types of arbitrariness that may occur in the explanation of a physical phenomenon (object, concept and theory arbitrariness), showing that none of them affects the EIA in any important way.[11] However, in addition to these three, we can point to another type of arbitrariness which, generally speaking, has often been present in the mathematical community. We will name it *isomorphic* arbitrariness. In effect, it is a mode of mathematical thinking which is expected and natural, a type of attitude for which every mathematician is prepared even during the undergraduate university education. When we analyze the content of a mathematical theory M_1, it is mathematically natural to wonder whether, perhaps, there exists another theory M_2 that would be isomorphic to the theory M_1.[12] If that is the case, then, theoretically speaking, every object, proposition, proof or explanation within the theory M_1 has its analogon in the theory M_2. It further implies that if a physical phenomenon P is explained by means of the object O_1 of the theory M_1, then it can be explained, with equal adequacy, by the corresponding object O_2 from the theory M_2. The choice of the alternative theory/object in this case does not depend on the physical phenomenon, but exclusively on the affinity of the researcher, or on some practical circumstances.[13] Which one of the objects, O_1 or O_2, is indispensable to the phenomenon P? None, according to Baker's intuition. Nevertheless, if we assume that there are no other objects which explain P, phenomenon P cannot be explained without at least one of these two objects. Therefore, they possess

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[11] For further information, see Baker (2009, 615-619).

[12] In other words, we will say that two theories (structures) M_1 and M_2 are isomorphic if there is a bijection between them that "preserves" all the relations and operations from the domain onto the codomain. If we would want to define isomorphic vector spaces within linear algebra, then we could do it in the following way:

An isomorphism between two vector spaces V and W is a map $f: V \to W$ that

1. is a correspondence: f is one-to-one and onto;
2. preserves structure: if $a, b \in V$ then $f(a + b) = f(a) + f(b)$, and if $a \in V$ and $k \in V$ then $f(k\,a) = k\,f(a)$.

[13] Molinini pointed out the role of pragmatic circumstances in the decision-making process when it comes to choosing a suitable mathematical theory for explanation of a physical phenomenon (cf. Molinini 2014). However, this text offers the examples from alternative theory (set theory) and Minkowski geometry, which are not isomorphic in the strictly formal sense.

a kind of *common* indispensability to P. A partial confusion created by this example proves a need for a more precise definition of indispensability.

On the other hand, the procedure of finding mathematical explanation of a physical phenomenon is methodologically similar to the procedure of finding mathematical explanation/proof of a mathematical phenomenon/proposition. In other words, extrinsic use of mathematical tools is methodologically similar to their use in intrinsic circumstances. When we deal with a proposition that should be proved in mathematics or, more realistically, when we have an intuitive sense of the correctness of a proposition, then we start from the already proved propositions and move towards the aimed proposition. There can be many proofs of this type and we could hardly ever state that we have reached their definite number.[14] Correspondingly, in the extrinsic conditions such as Baker's cicada example we do not have formal tools by which we could prove the indispensability of mathematical objects. Namely, how can we prove that there is no other explanation within the number theory or some other theory? To put it differently, in order to state a proposition on the indispensability of the prime numbers in the cicada example, we should have a proof of the impossibility of a different mathematical explanation, which is far from a trivial task. Generally speaking, if we know that at time t_1, O_1 is the only mathematical object (also the object of the theory M_1) used in the explanation of the phenomenon P, we cannot state the *absolute* indispensability of the object O_1 to the phenomenon P. In order to state such a proposition, we ought to prove that the phenomenon P cannot be explained at any other time t_n, $t_n > t_1$, of the development of mathematics, by no other object O_n (which would be an object of the theory M_n). Since at moment t_1 we cannot know the explanatory capacities of objects and theories which are to be created in future, we cannot hope for such a proof either. What makes sense, however, is a consideration of a *conditional* indispensability in this context, that is, an indispensability that would aim at establishing itself as such in relation to the objects of the mathematical theories defined prior to the moment of the consideration of indispensability. Does this make the situation simpler? It does, so far as it provides a clear domain of defined objects on which indispensability is to be examined. However, broadly speaking, is there a methodology by which we could

[14] For example, several proofs of the Fermat's little theorem are known today. A more extreme example provides several hundreds of proofs of the Pythagorean theorem. See Alkauskas (2009) and Loomis (1972).

precisely solve the question of the indispensability of the object that explains a phenomenon? Is there an algorithmic set of stages that would reveal with certainty that, for instance, there is no other mathematical object, taking into account all those defined so far, in all the theories, by which we could explain the cicada example? We are not in the possession of such a methodology at present, and the indispensability which we may attribute to an object is in this sense additionally conditional and relativized. The most we can say about an object is that it is indispensable to a phenomenon unless proved differently, which is a rather discouraging position from a researcher's viewpoint. Given this situation, it is far more pragmatic and reasonable to turn to more modest aims. One of these would be: a more precise definition of the concept of indispensability.

3. Is Molinini's definition suitable?

The first part of this paper has underlined, among other things, the importance of a more precise definition of the indispensability concept within the EIA, with the aim of re-examining the power of the Argument as one of the main supporting tenants of Platonism. Daniele Molinini was the one to make a decisive and welcome attempt at this, proposing a definition of indispensability. In effect, he offered an explicit definition of *dispensability* (henceforth 'D1'):

> A mathematical entity x is explanatorily dispensable to a scientific theory T if it is possible to find a theory T^* that:
> *(a)* does not employ the vocabulary of the mathematical theory M in which x is defined;
> *(b)* offers the same (or even more) explanatory power as T;
> *(c)* is empirically equivalent to T. (Molinini 2014)

We can notice that, when compared to Baker, the domain of the predicate *is (in)dispensable to* is more precise, at least in this definition. A mathematical object x is dispensable, or not, relative to a scientific theory T. In the first position of the predicate an individual mathematical object is implied, whereas the second position is occupied by an individual scientific theory. The intuition behind this definition is clear enough and it is similar to Baker's. Informally speaking, according to the definition, the mathematical object O is explanatorily dispensable to the theory *T* if it is possible to explain any phenomenon described by the theory *T* without *O*. Molinini provided several examples of the explanatorily dispensable mathematical objects, such as orthogonal matrices, Minkowski metric, set theory objects, etc. (see Molinini 2014), thus shattering the last hope that the EIA can be used to prove the proposition about the existence of all mathematical objects.[15]

D1 was expected to be the operative tool by means of which we could establish with certainty whether a specific mathematical object is indispensable to a specific scientific theory. Let us see if D1 reached this goal – a formalization of the concept which had been used non-formally beforehand, that is, if this formalization covered all the cases which we non-formally consider as dispensable. If we are to pursue Baker's intuition, we can state that the mathematical object *O* is explanatorily indispensable to the physical phenomenon *P* if and only if the phenomenon *P* cannot be explained without using the object *O* and its features. According to this, the mathematical object *O* is not explanatorily indispensable, that is, it is dispensable to the phenomenon *P* if and only if the phenomenon *P* can be explained without using the object *O* and its features. Therefore, intuitively speaking, a mathematical object is dispensable not only if there is an alternative to it when it comes to explaining a phenomenon, but also, as is trivially implied, if it does not explain a phenomenon at all. For example, Minkowski metric, as an object of the Minkowski geometry, is dispensable to a phenomenon of the theory of special relativity, known as FitzGerald-

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15 The problem of the so-called *weaker alternatives* in the explanation of phenomena is emphasized in Pincock (2012, 212-213). Claim *p* and claim *q* explain (individually) phenomenon *P*, with *p* being a stronger mathemathical claim than *q* (*q* follows from *p*, but not vice versa). If the explanatory power of claim *q*, when connected with the phenomenon *P*, is not lesser that that of *p*, it is not clear on what basis *p* would be preferred over *q*. For example, in the cicada case, let us assume that

p: prime periods minimize intersection (as compared to nonprime periods);
q: prime periods of less than 100 years minimize intersection.

According to Pincock we would be able to use *q* as an equally powerful explanation of the chosen phenomenom. In the context of the EIA, however, this quarantees existence of only those numbers smaller than 100, which is obviously an unacceptable consequence.

I am grateful to the anonymous referee who has brought my attention to this point.

Lorentz contraction.[16] We speak of dispensability in this case because in addition to the explanation of this phenomenon in which Minkowski metric is used, there is also an alternative – an axiomatization of the set theory by means of which the description of the contraction is acquired as a theorem.[17] Also, as for the Polish space, a general topology object, the same is true. Namely, on the basis of the available reference books, the Polish space is entirely unusable for explaining length contraction, which makes it dispensable to this phenomenon. As far as examples like these are concerned, D1 follows intuition. According to it, some of the objects of set theories, as well as Minkowski geometry, are not the only ones dispensable to the length contraction, but the same goes for the Polish space, being an object which does not explain it at all.

What we intend to suggest is that D1 has not covered all the intuitively dispensable objects. It is, therefore, too narrow. The problem here does not lie in the objects which explain a certain phenomenon, but for which there is an alternative explanatory object, neither in the objects which do not explain it but are part of the mathematical theory to which the object that explains the phenomenon does not belong. In these cases, D1 functions correctly. In other words, according to it, these objects are dispensable. The target of our attempts to show that this definition is not broad enough includes those mathematical objects that are dispensable on the basis of the criterion: "[it] does not explain a scientific phenomenon and belongs to the same mathematical theory as the object which does explain the phenomenon." Indeed, let us assume that x and y are objects of a mathematical theory M, the object x being enough to explain the phenomenon P of the scientific theory T, with no alternative of another object from another theory that could explain P, the object y included. Let us also assume that the object y does not explain any other phenomenon of the theory T. Intuitively, y is explanatory dispensable to the theory T since it is not used in any way to explain any of its phenomena. However, it is not dispensable according to D1, it is indispensable! How? In relation to y and T, the condition a) of the definition D1 was not fulfilled, because it is not possible to find a theory T^* which does not employ the vocabulary of mathematical theory M

16 It is a phenomenon in which the length of the body in motion is shortened, according to the precisely set formula, depending on the velocity of its motion in relation to the point of the observer.

17 On the proof of dispensabilty of the Metric and the use of the set theory in this case see Molinini (2014) and Andreka et al. (2007, 29-30).

in which x and y are defined and which fulfills the remaining two conditions of the definition. We can say that on the basis of D1, y is indispensable without taking merits for it, which is neither expected nor desirable. For example, if the numbers 13 and 17, or prime numbers in general, are indispensable objects of the number theory in the cicada example, then an object of number theory such as the previously mentioned Euler's function is altogether unusable for an explanation of the phenomenon and cannot therefore be intuitively indispensable.[18] Contrary to intuition, however, D1 gives it precisely that kind of status. Thus, D1 formally allows for a large class of objects to be considered indispensable, even though they are not intuitively experienced as such, which subverts the very purpose of defining.

Along the lines of these objections, we can propose a possible improvement of D1. It would suffice to alter only the initial part of the definition D1:

*A mathematical entity x is explanatorily dispensable to a scientific theory T iff either x does not explain any phenomenon described by the theory T, or it is possible to find a theory T * that:*

1. *does not employ the vocabulary of the mathematical theory M in which x is defined;*
2. *offers the same (or even more) explanatory power as T;*
3. *is empirically equivalent to T.*[19] (henceforth D2)

In addition, D2 includes all the types of the previously mentioned cases which we understood as dispensable and D1 did not treat them as such.

Despite the fact that the difference between D1 and D2 may appear as only technical and insignificant, it turns out that it changes the conception of the indispensability of the entire mathematical theory. Before expanding on this, let us refer to a notational remark. Hereinafter, due to reasons of brevity and

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[18] We assume here that the object in question is not used to explain another phenomenon which, along with cicada example, could be placed into a wider biological theory, such as, for instance, the cicadas' life-cycle theory or theory of the life-cycles of animals in general.

[19] In D2 we have not specifically differentiated between unexplanatory mathematical objects of the scientific theory *T*, depending on the fact if they do or do not belong to the mathematical theory whose object (possibly) explains a phenomenon of the theory *T*. A definition which would insist on such a sensibilty would probably be rather more complex and far more different from D1.

clarity, we shall refer to the mathematical object x which does not play an explanatory role in theory T at all as *trivially* dispensable. If an object x plays an explanatory role in theory T, but there is an alternative to it, another mathematical object y, then we shall say that x is *non-trivially* dispensable for T.

Another interesting novelty about D1 is that it considers the explanatory indispensability of a mathematical object to a scientific theory (or to a phenomenon of that theory) within the framework of a suitable mathematical theory, within which the object is defined. This approach seems correct, as the objects are defined by means of the vocabulary of the theory to which they belong. Also, the features of those objects are formulated in relation to other objects of the theory. Nevertheless, after the object-theory context in D1 was established, there is one thing which remained vague. Even though Molinini reserved the first position in the domain of *is dispensable* predicate for mathematical objects, by which he does not entail mathematical theories, he still speaks about dispensability of mathematical theory as well, asserting soon after that

> In fact, it says that dispensability of an entity is tantamount to the dispensability of the theory in which that entity is defined, and vice versa… (Molinini 2014)

This does not define dispensability of the theory at all. In this respect, Baker and Molinini take a similar position. The former employed the concept of an object's (in)dispensability in a non-formal manner, whereas the latter employed a theory's dispensability in such a way. If we attempt at questioning the justifiability of the above quotation, it ensues that we cannot treat the dispensability of a theory in a non-formal way either.

Every mathematical theory defines some mathematical objects.[20] At first, it may appear natural to state that a mathematical theory is dispensable to

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[20] An additional explanation should be provided at this point, which could have been done earlier, when the notion of *isomorphic arbitrariness* was introduced. I want to thank an anonymous referee for having brought my attention to this point. Namely, when we say that *every mathematical theory defines some mathematical objects* we are in effect referring to the theory-object relation which is common in mathematics. Theory *is composed* of objects, of their features and relations that exist among them. Objects are described by means of definitions and by means of propositions. When we say *objects* we refer to all basic and defined concepts that are part of a theory. For example, prime and composite numbers, as well as Euler's function are defined objects of number

a scientific theory *T* if all its objects are dispensable, that is, to state that a theory is indispensable if it has at least one object which is indispensable to the theory *T*. However, if we are to proceed in that way, then we would, for example, consider as dispensable the mathematical theory *M* which contains the objects *x* and *y*, both being non-trivially dispensable to the theory *T*, if, in that case, there are no objects outside the theory *M* which play an explanatory role in the theory *T*. It would mean that some phenomenon described by the theory T cannot be explained without the theory *M*, and, consequently, it would not be in accordance with the intuition that instructs us to state that the theory *M* is dispensable. For that reason, we need a new definition that would follow the usual intuition about the dispensability concept, also respecting the last particular case:

A mathematical theory M is dispensable to a scientific theory T if and only if for every object x of the theory one of the following conditions is fulfilled:
1. *The object x is trivially dispensable to the theory T;*
2. *The object x is non-trivially dispensable to the theory T and there is a mathematical object y which does not belong to the theory M, and which is non-trivially dispensable to the theory T* (henceforth D3).

This definition makes it clear that a mathematical object can fulfill only one of the set conditions. On the basis of this, we shall consider as dispensable only that theory in which all the elements are dispensable, with the exception that, if it is a non-trivial dispensability, we can find an alternative mathematical

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theory. Vectors and vector spaces are objects of linear algebra. Namely, both vector and vector space belong to the category of defined objects. Neuter element of the structure (N, +) is an object of the algebra, but that structure is itself also an object of the algebra. Indeed, both the neuter element and the structure (N, +) are also defined concepts. Thus, the world of mathematical objects is rather broad and composed of various entities, not unlike the biological world of which we all are parts. This complexity does not appear to be a reason for concern because it does not entail neither formal nor intuitive obstacles related to the analyses of the EIA. Let us mention that every mathematical object is observed in *the context* of some theory or, more specifically, in the context of some structure. For instance, the before mentioned neuter element can be observed in the context of a theory called algebra, but also in the context of a specific structure – groupoid (N, +). An arbitrary vector, for example $(a_1, a_2, \dots, a_n)$, $a_i \in R$, can be observed as an object in the context of a theory called linear algebra, but also in the context of a specific structure – n-dimensional vector space. See Drekalović (2015, 316-320).

object outside the theory *M*. We will be able, therefore, to explain the phenomenon described by the theory *T* without the theory *M*, which is in accordance with the intuition of dispensability. Eventually, it is obvious from the above given definition that we will state that a mathematical theory *M* is indispensable to the scientific theory *T* if and only if there is an *x* object of the theory *M* which is indispensable to the theory *T*, or is non-trivially dispensable but without a dispensable alternative which is not a part of the theory *M*.[21]

If we agree that the above definition describes to some extent the intuition of dispensability of a theory, let us examine from the formal standpoint the relation between a mathematical object and the theory which contains it. Obviously, dispensability of a mathematical theory and that of its object is not the same thing. It is far from that. To be more precise, according to D3, dispensability of a theory *M* entails dispensability of the objects of that theory. In other words, it cannot occur that a mathematical theory is dispensable to scientific explanations of a phenomenon and one of its objects is not, which trivially results from D3. On the other hand, on the basis of D3, generally, dispensability of an arbitrary object does not imply dispensability of the whole theory, with all its objects included. For example, some of the number theory objects, such as Euler's function, are dispensable to the cicada example according to D2, but that does not imply that the same goes when it comes to the entire theory. According to D3, as well as according to the expected intuition, number theory is indispensable to the mentioned phenomenon.

4. Conclusion

It seems that the EIA, in its present form, still cannot contribute to the strength of Platonism. This text has pointed to several reasons why that is the case. Firstly, the very formulation of the EIA contains elementary technical impreciseness related to the absence of appropriate quantifiers, which further

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[21] To put it more formally, a mathematical theory *M* is indispensable to the theory *T* if and only if there is a mathematical object *x* of the theory *M* for which two following conditions are required:

1. *x* is not trivially dispensable;
2. *x* is not non-trivially dispensable or there is no a mathematical object *y* which does not belong to the theory *M*, and which is also non-trivially dispensable to the theory *T*.

extends the impreciseness onto the ontological level. This form of the EIA leaves one of the main questions about existence in mathematics unresolved. Namely, it is not entirely clear, as the EIA has shown, whether the Platonists aspire to discuss only the existence of a limited number of mathematical objects, without dealing too much with the objects whose existence could not have been granted or, contrary to that, the EIA has a significantly larger aim to fight for the existence of the ultimately defined object of all the mathematical theories.

Why should we expect a solution to this Platonist position exactly from the EIA? Is it not too much to expect that, as an argumentative tool of a very short history, it can be employed to resolve a question which has remained open from the very beginnings of Platonism? There exists at least one reason why the great hope is invested in this argument. With the EIA's modal and syllogistic formulation, it has already been indicated that there are unquestionable tendencies towards stricter and almost formal explanation of the existence of mathematical objects issue. That kind of logical explanation is, at minimum, expected to offer a completely clear proposition about its field of reference – only some or all the objects. This field of reference cannot be seen in the EIA.

Molinini has reminded recently that the lack of precision is a general deficiency of the Argument, pointing to the desirability of an additional definition of the dispensability concept, which is essential to the Argument. His contribution is important not only because of the efforts to define the concept of dispensability on the basic level, but also because he underlined that it makes sense to consider dispensability of a mathematical object only in the context of the entire mathematical theory to which the object belongs, and not as isolated and independent from other objects of the theory. However, as we have seen, those attempts have in a sense also displayed some of their own drawbacks, both formal and fundamental. They have also remained incomplete. By incompleteness we refer exclusively to the intuitive approach to the concept of dispensability of a mathematical theory, even though in his criticism of Baker, Molinini has started precisely with the idea that the intuitive notion about an object's (in)dispensability should be reinforced with somewhat more formal approach. There is no reason why dispensability of a theory should not acquire the same treatment. We have drawn attention to a technical shortage of the definition D1, the reason why it does not encompass some of the trivially dispensable objects, and we have then proposed the definition D2, which surpasses that shortage. On the basis of that, as well as on the basis of the expected intuition, we have proposed the definition D3 of the dispensability of a theory. This has shown that (in)dispensability of a mathematical theory can by no means be the same thing as (in)dispensability of its arbitrary object.

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