Organon F 24 (1) 2017: 2-9

The Role of Disjunction in Some Alleged Non-Monotonic Inferences[1]

MIGUEL LÓPEZ-ASTORGA

ABSTRACT: Lukowski has argued that, if it is the case that there are actual non-monotonic inferences, they are very hard to find. In this paper, a representative kind of inference that is often considered to be non-monotonic is addressed. Likewise, certain arguments provided by Lukowski to demonstrate that that type of inference is not really non-monotonic are reviewed too. Finally, I propose an explanation of why, despite the fact that the arguments given by him seem to be convincing, it is usually thought that those inferences are not monotonic. In this way, I also try to account for the role that disjunction has in this issue and argue in favor of the idea that we can continue to suppose that the human mind does not ignore the essential requirements of classical logic.

KEYWORDS: Disjunction – inference – logic – monotonicity – non-monotonicity.

1. Introduction

Each theory claiming that the human inferential activity is logical must face the problem of the non-monotonic inferences. The motive of that is that classical logic is monotonic and it appears that, to solve the difficulties of those inferences, it is necessary either assuming the thesis that human

[1] Received: 2 September 2016 / Accepted: 15 December 2016

✉ Miguel López-Astorga
Institute of Humanistic Studies "Juan Ignacio Molina", University of Talca
Av. Lircay s/n, Talca, Chile
e-mail: milopez@utalca.cl

reasoning is not logical or looking for a logic other than the classical one, which can be thought to be non-monotonic. This is considered to be a fact and, from perspectives holding that the human mind does not work resorting to logical forms, it is often said that this problem is crucial for the theories stating that reasoning is about formal rules (see, e.g., Johnson-Laird, Khemlani, & Goodwin 2015, 201-202).

However, Lukowski's (2013) paper allows thinking about another possibility. According to him, while it cannot be maintained for sure that there are not real non-monotonic inferences, it is actually difficult to find an instance of inference that is so without a doubt. The truth is that his arguments seem to be absolutely convincing. Therefore, a justified question in this regard can be: if it is so hard to find non-monotonic inferences, why is the contrary a generally assumed idea?

To answer that question is the main aim of this paper. To do that, I will review an emblematic case of allegedly non-monotonic inferences and some arguments given by Lukowski (2013) in order to prove that they are not really non-monotonic. That kind is the one of the inferences related to "a reasoning increasing preciseness" (Lukowski, 2013, 67). Then I will show that the cognitive science literature provides results that enable us to understand why, in spite of his arguments, people tend to think that such inferences do be non-monotonic. Likewise, I will describe the relevance that disjunction and the logical rule of disjunction introduction (from now on, DI) have in this way and offer some commentaries supporting both Lukowski's (2013) theses and the idea that classical logic can be important in the human mind.

2. Monotonicity versus non-monotonicity

As it is well known, classical logic is, as mentioned, monotonic. As remembered by Lukowski (2013, 63-65), this basically means that, in that logic, if it is correct that $\{A\} \vdash \{B\}$, then it is also correct that $\{A \cup \Gamma\} \vdash \{B\}$. Thus, non-monotonicity is just the opposite. It refers to the situation in which, while that $\{A\} \vdash \{B\}$ is correct, that $\{A \cup \Gamma\} \nvdash \{B\}$ is so as well.

Clearly, as mentioned, this is a problem for any approach arguing that human reasoning is related to classical logic, since, if that idea were right,

all of our inferences would have to be monotonic. However, the point is that the non-monotonic inferences seem to be frequent. Lukowski (2013) reviews several examples in this regard, but one of them can be representative enough. That example is, as said, the one referring to the reasoning increasing preciseness.

Following Lukowski (2013, 65-67), the reasoning increasing preciseness is very usual in medical diagnosis contexts. Generally, in those contexts, the tests provide a number of results $\{\alpha_1, \ldots, \alpha_n\}$ that are linked to a number of possible conditions $\{\beta_1, \ldots, \beta_n\}$. Thus, if we based on his essential ideas without considering necessarily the order in which he presents his arguments, it can be said that the link can be understood as a deduction relationship, and that, if we assume these definitions:

$$A = \{\alpha_1 \wedge \ldots \wedge \alpha_n\}$$
$$B = \{\beta_1 \vee \ldots \vee \beta_n\}$$

it can also be stated that $\{A\} \vdash \{B\}$.

Nevertheless, the physicians often continue to carry out tests and hence obtain more results. Thus, $\{A\}$ can be transformed into a set $\{A'\}$ to which this identity corresponds:

$$A' = \{\alpha_1 \wedge \ldots \wedge \alpha_n \wedge \alpha_{n+1}\}$$

In this situation, it is absolutely possible that A' provides further information and that the new data lead the physicians to a new set of conditions $\{B'\}$ that can be defined as follows:

$$\{B'\} = \{\beta_2 \vee \ldots \vee \beta_n\}$$

As it can be noted, what has happened is that the new datum $\{\alpha_{n+1}\}$ has removed a possible condition $\{\beta_1\}$, and this can be considered to be a clear example of non-monotonic reasoning. The motive is obvious: while $\{A\} \vdash \{B\}$, $\{A \cup \alpha_{n+1}\} \nvdash \{B\}$. $\{A \cup \alpha_{n+1}\}$ is identical to A', and what can be deduced from it is not $\{B\}$, but $\{B'\}$, that is, $\{A'\} \nvdash \{B\}$, but $\{A'\} \vdash \{B'\}$.

In Lukowski's view, this is a really important process in medical contexts because it gives more preciseness progressively. Nonetheless, it is not clear that it describes a non-monotonic inference. His argument is straightforward, too (cf. Lukowski 2013, 67):

If $\{A'\} \vdash \{B'\}$, then $\{A'\} \vdash \{B\}$ too, as, in classical propositional calculus, $\{B'\} \vdash \{B\}$. In other terms, if $\{\alpha_1 \wedge \ldots \wedge \alpha_n \wedge \alpha_{n+1}\} \vdash \{\beta_2 \vee \ldots \vee \beta_n\}$, then $\{\alpha_1 \wedge \ldots \wedge \alpha_n \wedge \alpha_{n+1}\} \vdash \{\beta_1 \vee \ldots \vee \beta_n\}$ too, as, in classical propositional calculus, $\{\beta_2 \vee \ldots \vee \beta_n\} \vdash \{\beta_1 \vee \ldots \vee \beta_n\}$.

Therefore, the question is: if all of this is so evident, why this kind of inference is often considered to be non-monotonic? In my view, the key is in DI. I try to explain this in the next section with the help of the results about this last rule that are to be found in the cognitive science literature.

3. The problems of DI in human reasoning

Indeed, DI is the rule that is necessary to derive $\{\beta_1 \vee \ldots \vee \beta_n\}$ from $\{\beta_2 \vee \ldots \vee \beta_n\}$. As it is well known, DI is a rule that can be assumed as basic in a logic based on Gentzen's (1935) natural deduction calculus and formally expressed in this way:

$$\frac{p}{(\text{Ergo})\ p \vee q}$$

However, it is a controversial rule as well. The literature informs that people do not always tend to use it in a natural way. In fact, most of the time most of the people do not apply this rule (see, e.g., Orenes & Johnson-Laird 2012), and several current psychological theories about reasoning have explanations for this phenomenon. The case of the mental logic theory (e.g., Braine & O'Brien 1998a; O'Brien 2009; 2014; O'Brien & Li 2013; O'Brien & Manfrinati 2010) is especially relevant here, since it is one of the theories that continue to claim that human reasoning is based on logic nowadays. Nevertheless, this theory is empirical and does not consider all of the formal rules of classical logic to be essential schemata of the human thought, but only the rules of this last logic that are clearly used by people. This is important because, given that, as mentioned, the results reported in the literature show that individuals do not usually apply DI, the mental logic theory does not accept it as a basic rule, which means that it cannot be expected that people habitually use it.

Of course, if we assume the mental logic theory, this is an explanation of why the kind of inferences considered in the previous section is generally thought to be non-monotonic. If, to be aware that those inferences are actually monotonic, it is necessary to apply DI and people tend not to accept that rule, most of the individuals may not note the real logical nature of them and consider them to be inferences in which, when a new premise is added, what can be drawn is not exactly the same.

True, this option solves a problem. Nonetheless, it raises another one. If people do not often apply DI and that is a very important rule in classical propositional calculus, it is doubtful that the human mind works in accordance with that calculus. But solving this second problem is relatively easy. On the one hand, the fact that people generally use logic in their inferences does not mean that all of the inferences that can be made have the same difficulty level. Obviously, it can be assumed that some inferences and rules are harder than others. In addition, the mental logic theory also has the necessary machinery to respond to an objection such as this one. As explained, the theory proposes that there are a number of schemata that are not difficult and that, in all probability, people use whenever they have the opportunity. However, it is also possible to speak about sophisticated individuals that are able to make more complex inferences (see, e.g., Braine & O'Brien 1998b, 223). So, the possibility exists that certain individuals, who, for any reason, make logical inferences more easily than other people, use DI without difficulties. Thus, it can be said that the fact that we reason resorting to logical rules does not mean that all of us do that in the same way.

On the other hand, the proponents of the mental logic theory also claim that affirming the existence of a logic in the human mind does not necessarily imply stating that the only factor that plays an important role in the human intellectual activity is that logic (O'Brien 1998, 36-37). Thus, this very theory proposes, in the same way, that pragmatics is essential in reasoning too (Braine & O'Brien 1998d, 46ff) and that the mental logic is not absolutely incompatible with non-logical processes in the human mind (O'Brien 1998, 38). So, based on arguments of this kind, it can be said that ideas such as that the abductive inferences (that is, a kind of inference that is not admitted by classical logic) are used in medical diagnosis contexts as well (e.g., Pukancová & Homola 2015) are not a problem for the argumentation above either, since it can be thought that,

while it is obvious that the human cognitive architecture includes certain clearly logical schemata, it enables to resort, in some cases, to other mechanisms to obtain conclusions too. From this perspective, the difficulties related to the fact that people do not always apply any specific logic rule in a particular circumstance become relative, as the reasons for that fact can be many.

4. Conclusions

Lukowski (2013) also reviews other cases of alleged non-monotonic inferences. But, as far as I understand his arguments, his main idea is that most of them refer to monotonic inferences in which the conclusion does not change really due to addition of another premise, but because either a premise is changed by another one, which transforms the inference in other different inference, or the initial inference is not correct and the second one shows that. An example of this last case given by him is that of the "Tweety the ostrich" (Lukowski 2013, 60-70). It presents the situation in which, in principle, given that it is said that "Tweety is a bird" – {a}, it is drawn from it that "Tweety can fly" – {b}, since it is assumed that $\{a\} \vdash \{b\}$ (cf. Lukowski 2013, 69) Nevertheless, a problem can appear if, after that, it is stated that "Tweety is an ostrich" – {c}, as we would have to accept that, while $\{a\} \vdash \{b\}$, $\{a, c\} \nvdash \{b\}$ (cf. Lukowski, 2013, 69). The reason is evident: although Tweety is a bird, it is an ostrich too, and, as it is known, ostriches cannot fly. However, the Lukowski's explanation of why this is not a real non-monotonic inference is also clear. The key is just that a mistake has been made: "the error of generality" (Lukowski 2013, 70). Thus, as I interpret Lukowski's arguments, what really happens here is that it is not true that $\{a\} \vdash \{b\}$, since cases of $\{a \wedge \neg b\}$ are possible. In fact, the scenario in which Tweety is an ostrich is one in which we have $\{a \wedge \neg b \wedge c\}$. So, the inference is not actually non-monotonic, because the suitable deduction relationships are not $\{a\} \vdash \{b\}$ and $\{a, c\} \nvdash \{b\}$, but $\{a\} \nvdash \{b\}$ and $\{a, c\} \nvdash \{b\}$. Therefore, it seems that, from his point of view, accounting for the non-monotonic reasoning is really accounting for a type of very exceptional reasoning that is not common (or, if preferred, that is difficult to find).

The consequence of this is evident: the idea that a logical system based to a greater or lesser extent on frameworks similar to that of Gentzen

(1935) continues to be valid. However, this does not imply that there are not certain challenges to face. This paper has shown which two of those challenges can be. On the one hand, the cognitive theories holding that the human mind follows logical schemata must clarify when and under what circumstances the particular rules of inference are used, and when and under what circumstances they are not. We already know that DI is hard but not why. Likewise, it would be worth being absolutely sure about the difficulty of other schemata that have not yet been extensively studied and the reasons of it. On the other hand, it is also necessary to explain what being a sophisticated individual exactly means, the characteristics that are needed to be so, and, maybe, the variables that can have an influence on the fact that people become sophisticated as well. Likewise, given that not only logic takes action in our thought, it would be also desirable to clarify what the other factors or types of processes are actually and the particular circumstances under which those factors or processes can be used.

If works such as those indicated above are reviewed, there is no doubt that the mental logic theory, although, as said, it does not accept all of the schemata valid in classical logic, has made a significant progress in regard to the first challenge. It proposes even a reasoning program indicating the order in which the main schemata are usually applied (see Braine & O'Brien 1998c, 82-83, Table 6.2). Of course, further research is needed in this way, but it can be stated that there are already important conclusions obtained. In connection with the concept of logical sophistication, as far as I know, the situation is not the same. So the research on it is, to some extent, more urgent. Finally, although it is true that there are many studies about the non-logical machinery that the human mind can have (some examples have been cited in this paper), perhaps it would be interesting to continue to explore the exact situations in which logic has to be left and only other types of inferences can be made.

References

BRAINE, M. D. S. & O'BRIEN, D. P. (eds.) (1998a): *Mental Logic*. Mahwah: Lawrence Erlbaum Associates, Inc., Publishers.

BRAINE, M. D. S. & O'BRIEN, D. P. (1998b): A Theory of if: A Lexical Entry, Reasoning Program, and Pragmatic Principles. In: Braine, M. D. S. & O'Brien, D. P. (eds.): *Mental Logic*. Mahwah: Lawrence Erlbaum Associates, Inc., Publishers, 199-244.

BRAINE, M. D. S. & O'BRIEN, D. P. (1998c): The Theory of Mental-Propositional Logic: Description and Illustration. In: Braine, M. D. S. & O'Brien, D. P. (eds.): *Mental Logic*. Mahwah: Lawrence Erlbaum Associates, Inc., Publishers, 79-89.

BRAINE, M. D. S. & O'BRIEN, D. P. (1998d): How to Investigate Mental Logic and the Syntax of Thought. In: Braine, M. D. S. & O'Brien, D. P. (eds.): *Mental Logic*. Mahwah: Lawrence Erlbaum Associates, Inc., Publishers, 45-61.

GENTZEN, G. (1935): Untersuchungen über das logische Schließen I. *Mathematische Zeitschrift* 39, 176-210.

JOHNSON-LAIRD, P. N., KHEMLANI, S. & GOODWIN, G. P. (2015): Logic, Probability, and Human Reasoning. *Trends in Cognitive Sciences* 19, No. 4, 201-214.

LUKOWSKI, P. (2013): Is Human Reasoning Really Non-Monotonic? *Logic and Logical Philosophy* 22, 63-73.

O'BRIEN, D. P. (1998): Mental Logic and Irrationality: We Can Put a Man on the Moon So Why Can´t We Solve Those Logical Reasoning Problems? In: Braine, M. D. S. & O'Brien, D. P. (eds.): *Mental Logic*. Mahwah: Lawrence Erlbaum Associates, Inc., Publishers, 23-43.

O'BRIEN, D. P. (2009): Human Reasoning Includes a Mental Logic. *Behavioral and Brain Sciences* 32, 96-97.

O'BRIEN, D. P. (2014): Conditionals and Disjunctions in Mental-Logic Theory: A Response to Liu and Chou (2012) and to López-Astorga (2013). *Universum* 29, No. 2, 221-235.

O'BRIEN, D. P. & LI, S. (2013): Mental Logic Theory: A Paradigmatic Case of Empirical Research on the Language of Thought and Inferential Role Semantics. *Journal of Foreign Languages* 36, No. 6, 27-41.

O'BRIEN, D. P. & MANFRINATI, A. (2010): The Mental Logic Theory of Conditional Proposition. In: Oaksford, M. & Chater, N. (eds.): *Cognition and Conditionals: Probability and Logic in Human Thinking*. Oxford: Oxford University Press, 39-54.

ORENES, I. & JOHNSON-LAIRD, P. N. (2012): Logic, Models, and Paradoxical Inferences. *Mind & Language* 27, No. 4, 357-377.

PUKANCOVÁ, J. & HOMOLA, M. (2015): Abductive Reasoning with Description Logics: Use Case in Medical Diagnosis. In: *Proceedings of the 28th International Workshop on Description Logics.* Athens.