On Tichý's Attempt to Explicate Sense
in Terms of Turing Machines

PAVEL MATERNA[1]

ABSTRACT: In Tichý (1969), it is shown that semantics of natural language can be pursued procedurally. Tichý supported his argument by defining elementary functions of logic (truth functions, quantifiers) using Turing machines and attempting to define the sense of empirical expressions using a simple semantic version of oracle. From the way how Turing machines and later constructions are defined it follows that even the sense of empirical expressions can be successfully handled but that the sense and denotation can be in principle effectively obtained while the actual value at the actual world can be, of course, never computed. The present paper comments on this attempt and compares the Turing machines argument with the possibilities given by TIL constructions. Turing machines guarantee the effective character of computing while the constructions do not, but expressive power of constructions is incomparably stronger, not only because Tichý's possible worlds from 1969 are atemporal: they define essentially 1st order operations and can be reinterpreted as one possible world enjoying (discrete) temporal changes. Both the TM conception and the "constructivist" one know that the question "which possible world is the actual one" cannot be ever answered by effective (computational) methods and their analyses of empirical expressions are therefore compatible.

KEYWORDS: Oracle – possible worlds – procedure – sense – Turing machine.

✉ Pavel Materna

Institute of Philosophy, The Czech Academy of Sciences
Jilská 1, 110 00 Praha 1, Czech Republic

e-mail: materna@flu.cas.cz

1. Tichý's explication of Frege's *Sinn* in 1968

Frege's ingenious idea from 1892 of mediating the denotation (*reference*) by *sense* (as "the mode of presentation") was baptized "sense" but we know that Frege never *defined* the sense. The idea itself was however guessed to be sound. Nowadays it should be clear that to solve *semantic* problems of a natural language without using some notion essentially similar to Frege's *sense* means to replace semantic criteria by *pragmatics.* This is what Quine did in his famous (1953). His influence was strong, and soon one could read formulations like "Quine teaches us that…" or "Quine refuted the claim that …" instead of more sober phrasings like "According to Quine…" or "Quine refused the claim that…". Quine's argumentation was interesting and we have to quote a key point of it to appreciate Tichý's *semantic turn.*

In Quine (1953), he says:

Once the theory of meaning is sharply separated from the theory of reference, it is a short step to recognizing as the primary business of the theory of meaning simply *the synonymy* of linguistic forms and *the analyticity* of statements; *meanings* themselves, as obscure intermediary entities, may well be abandoned. (Quine 1953, 22; italics mine, P.M.)

Terminological remark: It frequently happens that *meaning* is used instead of *sense.* As for *denotation*, Frege has *Bedeutung*, Church *denotation*, Geach and Black *reference.* Here we will use *denotation* and we will show that *reference* differs from *denotation* when empirical expressions are analyzed. Further, Quine identifies *meaning* with *intension* (cf. Quine 1953, 21).

From the very beginning Quine believed that the only logic deserving this name is just *extensional* (we could say "denotational") logic and even speaking about intensions was suspect for him. Therefore the term *meaning* or *sense* as an indication of something, which could threaten the idea of the extensional monopoly, was characterized as an "obscure entity". Thus we should, according to Quine, try to logically explain some phenomena that are allegedly defined by means of *meaning* and to show that such explanations necessarily fail.

The way Quine argues to show that synonymy and analyticity cannot fulfil their role is well-known. Don't forget however that Quine does not accept the possibility to use *meaning* as to solve the problems that make the task of satisfactorily defining the critical notions not feasible (see Materna 2007).

Now we can compare Quine's criticism with Tichý's (1968/2004). Tichý then did not know Quine's work (if he had known it he would surely have quoted the respective article. The world logical literature could not be sold in Communist Czechoslovakia then), but the text I will now quote looks like a comment to Quine:

> In current logic there is a strong tendency to define the sense by means of synonymy or analytical identity of expressions. It stems from the assumption that the relation of synonymy or analytical identity is definable without the notion of sense. "… *It follows a description of the way it can be done*." This approach is formally correct, but from the semantic-content point of view we can object that this method of defining is quite opposite to our intuition. "*It follows again a description of that way and the respective critique*." Hence in both cases defining the sense by means of the relation of analytical identity is either to turn over the natural logical sequens of these notions, or to fall into a circular definition.[2] (Tichý 1968/2004, 81)

Tichý's conviction had to be supported by such a notion of meaning (here also denoted by *intension*) that would be compatible with the way the term *meaning/sense* is used in the Natural Language and possesses the (evidently Fregean) property

FP   *Two senses may concern one and the same object*

or in the more terminologically precise variant

FP′   *Two expressions that express distinct senses may denote one and the same denotation.*

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[2]   Here the pagination concerns the translation of Tichý (1968) in Tichý (2004).

Tichý has emphasized that the *classical* conception of sense (in particular the Aristotelian theory), although inadequate from the viewpoint of the contemporary logic, was closer to the truth that "the notion of intension[3] logically precedes the notions of truth, analyticity and synonymy, and not vice versa" than the contemporary attempts, so that "it is just necessary to replace the oversimplified classical family of qualities by a construction that would satisfy the requirements of modern logical analysis" (Tichý 1968/2004, ibidem).

The key notion which led Tichý to his solution was mentioned at the outset and in the title of the Czech paper from 1968: *Sense and procedure.* Here he writes:

> [i]t is easy to see that, taken in an abstract way, the relation between sentences and procedures is of a semantic nature; for sentences are used to record the results of performing particular procedures. (Tichý 1968/2004, 80)

In the following text, which was intended as a popular Czech version of Tichý's theory, the author introduces basic concepts of this theory, viz. *universe*, *intensional base*, *empirical system*, *Turing machines*, *procedures*, *possible worlds*, *languages as unities of primitive symbols and interpretation*, and *synonymity* and *analyticity*, so that *logical truth* and *analytical truth* can be defined and such "obscure entities" like *sense* or *meaning* get exact definitions.

2. The exact formulation in (1969)

In *Studia Logica* paper (see Tichý 1969/2004), Tichý has published a precise version of what has been articulated in Tichý (1968/2004). One basic term is characterized at the very outset, the term, which Quine characterized as an "obscure one": the term *sense.* Tichý says: "The sense of an expression is an entity linking the expression with its denotation" (Tichý 1969/2004, 96). A most important refinement of this formulation follows: to know the sense of an expression "does not mean actually to know its

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3   Here Tichý means by *intension* a family of features, cf. Aristotelian definition.

denotation but to know how the denotation can be found, " so that "it seems to be natural to conceive of concepts as procedures" (Tichý 1969/2004, 97).[4] Applying a procedure to an input data consists in performing various transformations of this data according to a fixed program. Atomic units of these transformations are *steps*, either *autonomous* (the result depends just on the outcome of the foregoing steps) or *empirical* (the result depends on the state of the external world). So an *empirical system* which underlies each language consists of the external world, i.e. (finite) *universe of discourse* containing just bare individuals, and its state that is given by applying the members of *intensional basis* consisting of elementary tests that can be applicable to individuals or couples of individuals etc. The assumption is that all the members of the intensional basis are intensionally independent, and that the number of the available elementary tests is finite, which guarantees that as a conceivable state of the universe can be regarded any combinatorial possibility of the results of applying all the tests (from the intensional basis) to all the individuals (couples of individuals etc.). These possibilities can be called *possible worlds (with respect to) the empirical system.*

An empirical system is according to these definitions in a good sense *finite*. The finite character (to be argued for in the next paragraph and on pages 97, 98 of Tichý's 1969/2004 paper) makes it possible to apply Turing machines and, thereafter, to compare this finite version with the contemporary version of TIL.

3. The finite character of the 'empirical system' from 1969

Rewording what Tichý says in (1969/2004, 97) we get

- K the cardinality of the Universe of discourse
- S the greatest number s such that there are s-ary members of the intensional basis

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[4]   Here we can state that the future procedural theory of concepts is suggested, see, e.g., Duží, Jespersen & Materna (2010).

P_0, P_1, …, P_S the number of 0-ary, …, S-ary tests in the intensional basis
(the sum of the particular numbers)

The sequence $K\,S\,P_0\,P_1…P_S$ of natural numbers is called the *dimension*
of the system. It makes it possible to define the length of the sequence (W)
that corresponds to one realization of applying the tests from the inten-
sional basis. We define this sequence (W) as follows (cf. Tichý 1969/2004,
98):

> $W^{qp}{}_r$ *is 0 (1) if the outcome of the q-^{ary}p-^{ary} test from the intensional
> basis applied to the r-th ordered p-tuple of individuals from the universe
> is positive (negative) in the possible world W.*

Clearly, the length *N* of (*W*) equals the

Sum (Σ) of $P_i\,K_i$ from *i = 1* to *S*

so that the *number of possible worlds defined in this way equals* 2^N. Inter-
esting news: Having an empirical system we have got, true, many possible
worlds but *finitely many of them*.

We will see, however, that some important properties of an empirical
system have to be sacrificed to get a finite number of possible worlds. An-
yway, an easy way how to represent an empirical system by a set of natural
numbers less than *N* has been shown.

Now we will first comment on some important consequences of Tichý's
conception of possible worlds in the years 1968 and 1969.

4. The actual world

Tichý states (see Tichý 1969/2004, 98) that there is a possible world
having a privileged status: the so-called *actual world*. The actual world
will be denoted by *W**. It is of the form (*W*), where instead of $W^{qp}{}_r$ *is 0
(1) if the outcome of the q-^{ary}p-^{ary} test from the intensional basis ap-
plied …* we have *basis actually applied…* (emphasis mine, P.M.). Here a
very important claim follows: "It goes without saying that within seman-
tics we have no way of telling which of the possible worlds with respect

to a given empirical system is the actual one, since this is an (or rather *the*) empirical question" (Tichý 1969/2004, 98).

This statement is of key importance. It can be argued for in more details and we will therefore incorporate into the present paper a brief *intermezzo*.

Intermezzo: TIL on the semantics of empirical expressions. Tichý's Transparent Intensional Logic whose contemporary version can be found in Duží, Jespersen & Materna (2010) developed from Tichý's early intuitions and his papers from 1968 and 1969. Here we would like to show some details of arguing for the foregoing claim concerning the actual world. We will apply the contemporary TIL to the way Tichý talks about possible worlds in 1968, 1969. Thus what does TIL say about semantics of empirical expressions:

The sense of an empirical expression is always a construction that constructs a non-trivial intension. The type of an intension is $((\alpha\tau)\omega)$, α any type, τ for time moment, ω for possible worlds; $\alpha_{\tau\omega}$ is used as an abbreviation. Thus an empirical expression denotes a *function* (*viz. of possible worlds and times*), which means that *it never denotes an actual object.* The expression *the Pope*, for example, denotes the function $\iota_{\tau\omega}$ (an individual role/office), never a concrete individual that happens to be the Pope. Applying this function to W^* we would get the actual Pope, e.g. John Paul 2th in the year 1980 *if the W^* were a concrete possible world*, *but the type of W^* actually is $(\omega\omega)$*, *since the w*, *where W^* is actual*, *is*, *of course*, *w*. Thus W^* is again a function, and we cannot know which its value is: the logic itself does not give us a criterion – as a semantic question it is beyond deciding.

The vain effort to 'construct' the transition from the denotation to *reference* can be explained just by this theory of the role and character of the *actual world.* The possibility to distinguish denotation and reference has been exploited only by TIL, where *denotation* is determined by the sense and is thus an analytic part of Frege's scheme whereas *reference* is defined as the (contingent!) value (if any) of the denotation of an empirical expression in the actual world. This distinction holds true only when empirical expressions are analyzed. (See Duží, Jespersen & Materna 2010, 176-187, 13-14.) *End of Intermezzo.*

5. The empirical system from 1969 is atemporal

The greatest distinction between the 1969 paper and the TIL version from 1988 and 2010 consists in the fact that the former unlike the later is *atemporal.* Thus the length of the 'dimension' can be always a natural number. It means however that Tichý's empirical system corresponds at most to one temporal unit (moment) and represents the answer to the question "*what can happen during one moment when the given intensional basis is applied to the members of the given Universe*". Our intuition has it however that one and the same possible world as a consistent combination of the outcomes of 'tests' can enjoy temporal changes so that it is rational to respect not only the procedures that lead to transitions to a distinct possible world but also changes that occur within one and the same possible world, namely temporal changes. The atemporal conception has been abandoned by Tichý in his (1988; see especially p. 186), and in the following pages the temporal conception is analyzed and argued for. Observe further Tichý (1980/2004, 373), where Tichý already works with temporal analyses.

We can see that even in very simple cases the analysis (for example of the sentence "It rains in Prague" would be counterintuitive if the type of propositions were $(o\omega)$. Clearly, this atemporal decision is incompatible with our most basic intuition: in one and the same possible world we can state once T, once F, our system is inconsistent. As soon as the type is $((o\tau)\omega)$, or $o_{\tau\omega}$ for short, our intuition is restored, every possible world leads to a function from time to α (here o). So *give me the possible world and the time and I will say whether it rains in Prague*.

As soon as *time* is accepted the idea of a finite empirical system has to be abandoned, at least when time is continuum, which is the case of TIL, where τ are *real numbers*. The papers from 1968 and 1969 remain to be finite in the sense explained above. The fact that N has been defined in that sense means that what has been actually defined is surely different from our intuitive notion of a class of possible worlds: we have already guessed that those definitions determine rather a class of temporal changes within one possible world. Let any such class be called Nw_i where $i > 0$ and N is Sum (Σ) of $P_i K_i$ from $i = 1$ to S, where K is a constant universe and the values of S and P_i are dependent on w_i. Adding the temporal dimension means that a class of possible worlds arises as a class the members of which are Nw_i as interpreted in this way.

6. Procedures

Tichý's *empirical system* has been type-theoretically defined: we have got *universe* with *individuals* as its members, truth-values (0, 1), so that 1st order relations are definable. Besides, possible worlds are defined as consistent combinatorial possibilities w.r.t. outcomes of 'tests'. Before a 'formal language' is introduced a kind of Turing machines is described whose last quadruplet represents the state of the external world ("oracle"), i.e. the world *W*. (See the chapter 4.) The actual world w.r.t. the given empirical system is just the world that is the outcome of an actual Turing computation). An illusion may arise that, therefore, Turing machines could compute the value of the denotation of an empirical expression, which would contradict to the important claim that within semantics this is impossible (ch. 4). Simply: Which world is actual *in the world W?* Clearly, the world *W*. The *denotation* of the empirical expression "the actual world" is the function of the type $(\omega\omega)$. A *reference* is the contingent value of this denotation in the actual world. This reference can be never reached by a computation *ex definitione*: "the actual world" is an *empirical* expression, i.e., it is not a semantic expression.

Anyway, Tichý introduced in 1969 Turing machines to show that the *sense* of an expression is a *procedure* and he demonstrated the possibility of an exact explication of this notion. The 'mature' stage of TIL has instead defined the notion of *construction* (see Tichý 1988; Duží, Jespersen & Materna 2010). Constructions are no more atemporal and make it possible to procedurally define hyperintensionality (unlike Montague). In Tichý (1969/2004) procedures have been explicated in terms of Turing machines and the expressive power of this explication has been very weak in comparison with constructions. The choice of Turing machines meant that the *effectivity* of the respective computation was guaranteed. The transition to *constructions* lost this guarantee. As Tichý says in (1986/2004):

But not every construction is an algorithmic computation. An algorithmic computation is a sequence of *effective* steps, steps which consist in subjecting a manageable object (usually a symbol or a finite string of symbols) to a feasible operation. A construction, on the other hand, may involve steps which are not of this sort. An application of any function to any argument, for example, counts as a legitimate

constructional step; it is not required that the argument be finite or the function effective. Neither is it required that he function constructed by a closure have a finite domain or be effective. As distinct from an algorithmic computation, a construction is an *ideal* procedure, not necessarily a mechanical routine for a clerk or a computing machine. (Tichý 1986/2004, 613)

Let us return to Tichý's *empirical system* from 1969. Here all relevant notions are defined in terms of which the semantics of the syntactically simple languages is definable, namely languages with a fixed syntactic basis *B*, where *I* is *interpretation* that assigns the primitive symbols of *B* with their *senses* given by the empirical system *E*. Then the *value of* a well-formed formula or a term is given for a given *valuation* and *truth* of a well-formed formula, logical truth and analytic truth are safely defined.

(A linguistic remark: When Tichý *quotes* Frege's paper he speaks about *Sinn and Bedeutungi* but he translates *Bedeutung* as *meaning* (which corresponds to a translation from German). We should not forget however that in the contemporary TIL literature Frege's idea is better understood if *Sinn* is what Frege would call *meaning* and *Bedeutung* would be interpreted as *denotation.*)

Constructions are much more complicated and are not limited by 1st order, they are not atemporal and are able to solve more important problems due to greater expressivity but effectivity is not guaranteed (see above). All the same some essentially new approach to analysis of natural language can be stated in the 1968 and 1969 papers.

Summarizing: Unlike Quine and most logicians (not only) then Tichý does not believe that the only proper logic is a set-theoretical logic, and from the very beginning he applies his logic to analyzing natural language on the assumption that the empirical expressions do not denote actual objects but offer the possible ways how to express their *senses*. The senses of empirical expressions are given by *tests* and registered in an *intensional basis.* The *interpretation* of the primitive symbols assigns *senses* rather than denotations to them.

A comparison with *constructions* in TIL in this respect is interesting. The *sense* of an expression (in the given language) is always the *construction* which is the result of logical analysis. Thus intensions as functions from possible worlds are what is *denoted* by the respective (empirical)

expression i.e. what is *constructed* by the respective sense. In the 1968, 1969 papers it seems that senses are very similar to this 'mature' conception. Indeed, first we read: "*The sense of an expression is an entity linking the expression with its denotation*" (Tichý 1969/2004, 96). This is just the sound idea underlying Frege's *Sinn*. Now Tichý refines this idea as follows: he asks, what it means to know the sense, for example of the word "taller", and says: "*to know a method or procedure by means of which the relation* (here "taller") *can be identified.*" Atomic units of the respective transformations are *steps* and we can distinguish *autonomous* and *empirical steps:* the former are as their outcomes dependent just on the preceding steps, the latter are dependent on the state of the universe. Thus if the outcome of a procedure depends on the state of the universe then the respective procedure corresponds to an empirical expression. Thus the expression denotes what the outcome of the respective procedure is. So we can have a (rough) translation of the language of 1969 to the language of TIL:

1969	**1988**
Concept is a procedure	Concept is a procedure
Procedure is a Turing machine	Procedure is a *construction*
Outcome of (T, n)	The entity (if any) constructed by the construction

In particular: Outcome of (T, n) for empirical transformations is a nontrivial *Intension*: every outcome of a (T, n) is a (maybe nullary) *function.*

7. Conclusion

1. Tichý's work from 1968 to 1988 shows a nice example of the development of a basic intuition to its fully explicated form.

2. The main idea is explicating the intuition concerning the *meaning/sense* of NL expressions as a *procedure* (1968 a Turing machine, 1988 a construction).

3. Instead of simply accepting the standard conception of formal language (syntax + axioms and rules + interpretation) Tichý exploits Turing machines as modeling the sense (together with an intuitive conception of NL).

4. What is added or what modifies the 1968/1969 theory to get the mature stage of TIL is, essentially, temporality and higher-order types. This makes it possible to get a *procedural theory of hyperintensionality.*

Acknowledgement

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