Denotation and Reference

Pavel Materna

Abstract: The terms denotation and reference are commonly used as synonyms. A more fine-grained analysis of natural language as offered by TIL[1] shows that we can distinguish these terms in the case of empirical expressions. The latter are shown to denote non-trivial intensions while their reference (if any) is the value of these intensions in the actual world.

Keywords: Transparent intensional logic (TIL), denotation, reference, meaning, intensions.

1 Terminological Problem?

The well-known Frege's semantic triangle opened a series of comments and analyses that among others tried to explicate the central notions of this triangle, i.e. notions expressed by *Sinn* and *Bedeutung*. These two terms were themselves used by Frege as a kind of explication since in German they possess various semantic values. Frege however did not define what *Sinn* and *Bedeutung* should mean. To say that Sinn is "die Art des Gegebenseins", i.e., "the mode of presentation"(Geach – Black 1952, 57) is an important characterization, which is however highly indeterminate, as we can state when reading so many later attempts at explication. As for Bedeutung the situation is still worse. Frege gives us examples of *Bedeutung* in his (1892) and says some important details concerning *Bedeutung* of 'concept words' in Gabriel (1971, 25 – 34). What is somehow determinate can be derived from Frege's motivation as contained in the outset of (1892): it

1 See Duží – Jespersen – Materna (forthcoming).

concerns the relation between sense and 'reference' and can be formulated as in Gabriel (1971, 34):

> Die Logik muss sowohl von Eigennamen als auch vom Begriffsworte fordern, dass der Schritt vom Worte zum Sinne und der vom Sinne zur Bedeutung unzweifelhaft bestimmt ist.

(We will however see that on prevailing contemporary interpretations this demand is unrealizable so that a different interpretation is necessary.)

The real problems with the indeterminacy of Frege's terms necessarily influenced terminological problems. See "Glossary" in (Geach – Black 1952, ix, ad *bedeuten, Bedeutung*):

> The natural rendering of these words would be 'mean' and 'meaning'; this rendering is actually required for their occurrence in German works quoted by Frege, and for his own use of the words when alluding to such quotations. But 'meaning' in ordinary English often answers to Frege's *Sinn* rather than *Bedeutung*… Philosophical technicalities, like 'referent' or 'denotation'… would give a misleading impression of Frege's style.

Tichý in his seminal monograph *The Foundations of Frege's Logic* (1988) sees the disambiguated Frege's triangle as follows (p. 103):

> Once the Fregean notion of sense is disambiguated… into that of a presentation and that of a construction of a presentation, Frege's account of meaning becomes four-fold. An expression expresses its sense, which constructs a presentation, which in turn determines the referent.

In the present paper we decide to use following terms: *meaning* for what Frege obviously intended to call *Sinn*, then (ignoring for good reasons Geach-Black's warning) *denotation* for Frege's *Bedeutung*,[2] and *reference* for what will be distinguished from denotation in the case of empirical expressions. Mill's term *denotation* as confronted with *connotation* can remind us of the contrast between *extension vs. intension* but is heavy-laden by the traditional (rather psychologistic) vocabulary.

[2] Similarly as A. Church in his (1956).

2 Expressions Denote the Output of Meaning (if any)

If *denotation* and *meaning* were semantically independent terms then any explication would be dubious due to the fact that both terms are rather cloudy. Fortunately Frege's exposition of his motivation makes one thing clear: Meaning should be a way to denotation, i.e., *the meaning of an expression E should be a way to the denotation of E.* This is obviously the core of Frege's idea of splitting the semantic value of an expression into meaning and denotation. Moreover, according to one of our quotations above *the step from the meaning to the denotation should be unambiguously ("unzweifelhaft") determined.*

We cannot proceed further without attempting at an explication of *meaning*. Two demands have to be fulfilled:

a) The link *expression* → *meaning* should be unambiguous, i.e., independent of empirical facts, i.e., *a priori.* (Frege's intention is incompatible with any conception of meaning which would tolerate more meanings of an expression.)
b) The link *meaning* → *denotation* should be unambiguous (in the same sense).

Now we will show that the demand a) can be easily fulfilled as soon as we get rid of some empiricists' prejudices. The fulfillment of the demand b) is a much more complicated task.

3 Linguistic Convention and Meanings

First of all, to realize a *logical* analysis of natural language (LANL) we have to be aware of the fact that we consider *the linguistic convention* that associates sequences of signs/sounds of the given language with meaning *to be already given*. Tichý puts the point succinctly in a 1966 paper:

> We assume, of course, a normal linguistic situation, in which communication proceeds between two people, both of whom understand the language. Logical semantics does not deal with other linguistic situations.
> (Tichý 2004, p. 55, n.1)

Thus it is not our task to investigate the circumstances of origin and development of this process of arising meaningful expressions (which is done by empirical linguistics). Therefore, neither it is our task to determine *norms* for what the expressions of the given language *should* mean: if we are told by a language user A that the expression *e* means *m* while the user B claims that *e* means *n,* where $m \neq n$, we are ready to offer *logical consequences* of both the claims and let A and B decide what is better.

On this assumption we can defend the view that the meaning[3] of an expression *e,* as given by the linguistic convention, is independent of empirical facts: since we do not consider the linguistic convention to be an empirical fact we identify *experience* with *extra-linguistics experience*, and to know the meaning of an expression, i.e., to *understand* the expression, we surely do not need to know extra-linguistic facts. (Otherwise we would never understand any expression.)

4 Structured Meaning

To argue that the link *expression* → *meaning* is independent of experience it was sufficient to characterize meaning as what is ascribed to an expression by the linguistic convention. It was not necessary to be more specific as concerns the character of meaning.

To inspect the link *meaning* → *denotation* we cannot manage without such a more specific characteristic of meaning. Should we suppose that meanings could be *intensions* in the sense of P(ossible) W(orld) S(emantics)? Despite of the wide-spread conviction of many analysts of Frege's philosophy we can show that PWS intensions cannot serve as meanings:

PWS intensions are *functions*, they are mappings from the logical space (possible worlds) mostly to chronologies (functions from time moments) to something (truth-values, classes, relations, time moments etc. etc.). As mappings they are *simple*: they do not possess a structure, let alone a structure that would correspond to the structure of the given expression. Further, mathematical expressions would be meaningless since no PWS intensions can be connected with them.

3 Here and everywhere we assume that the language is given.

We can generalize: *no set-theoretical entities can play the role of meaning*. Meanings should present a denotation (if any), but as Zalta in (1988, 183) rightly says:

> Sets… are not the kind of thing that would help us to understand the nature of presentation. There is nothing about a set in virtue of which it may be said to present something to us.

Further: If the meaning of the given empirical expression were an intension then the denotation of this expression would have to be the value (if any) of that intension in the actual world (+ time). This actual value of an intension is however not determined by the intension: we need experience because the linguistic convention (and so the meaning) cannot know which of the possible worlds the actual one is. It cannot know, e.g., which person will be the Pope in 2008, which town will be the capital city of Poland at that time etc., while the conditions to be fulfilled by an individual who is the Pope, or the conditions that an individual must fulfill to be the capital city of Poland are unambiguously and independently of any empirical facts given due to the meaning of the respective expressions. So the way to the denotation would not be *unzweifelhaft, a priori.*

But what does it mean 'to be structured'? Cresswell was probably the first who began talking about 'hyper-intensionality' (1975) and 'structured meanings' (1985). He (similarly as Kaplan) proposed to represent structured entities *via* ordered tuples. Tichý in (1994, see 2004) and Jespersen in (2003) have shown that the 'tuple-proposal' captures only one feature of being structured: the members of the tuples can be seen as meanings of the particular subexpressions of the respective expression, but the meaning of the expression itself cannot be reduced to a sequence of the meanings of the subexpressions. After all, tuples are also set-theoretical entities.

The first logician who discovered that the non-set-theoretical alternative to the set-theoretical attempts of explicating meanings is a *procedural semantics* was Pavel Tichý. In his (1968, see 2004, 80) he says:

> T]he relation between sentences and procedures is of a semantic nature; for sentences are used to record the results of performing particular procedures. (2004, 80)

In the same article, as well as in (1969), he represented (abstract) procedures by Turing machines. Later,[4] as the founder of Transparent intensional logic (TIL), he defined *constructions,* exploiting λ-calculus and interpreting λ-terms objectually, as extra-linguistic abstract procedures.

Here we do not reproduce exact definitions of constructions. Suffice it now that we compare the λ-term $\lambda x\ (x + 1)$ with the construction $\lambda x\ [{}^0{+}\ x\ {}^0 1]$, $x \to \mathbf{N}$: While the term is interpreted as the Successor *function* the construction is the (here) algorithmic *computation* whose particular steps are given by the instruction above (something like "add the natural x to 1 and abstract over x") and *whose output is the function* called Successor. Whereas the term is interpreted as a structure-less mapping, the construction ("computation") contains particular steps that, by the way, correspond to particular subexpressions of the expression *add(ing) 1 to a natural number.*

Now we can solve the problem that motivated Frege to his distinguishing between meaning and denotation: How come, Frege wonders, that a true sentence of the form $a = b$ can be informative unlike a sentence of the form $a = a$, when a, b in the former sentence denote one and the same object?

The attempts of some interpreters of Frege to take PWS intensions as being Frege's intended senses can be understood (albeit not accepted, as we have shown above) since one of Frege's examples of sentences of the form $a = b$ is the famous *Abendstern* = *Morgenstern* example. The first example adduced in 1892 is however the example with triangle medians, where no intensions can occur. To take another example of this kind, which is simple and widely applied, consider the sentence $3 + 5 = + \sqrt{16}$. Denotations: Left side – 8, right side – 8. Of course, the sentence does not say that 8 = 8. We can say with Frege, that the left side presents 8 in another way than the right side, i.e., the 'sense' (we say 'meaning') of the left side expression differs from the sense of the right side. So what is the meaning of the left side, of the right side?

Once more: the sentence does not say anything about the number 8. It claims however that something is identical with something, only

[4] Especially in his monograph (1988).

this something is not 8. A paraphrase of what the sentence says could be:

> The outcome of *the procedure*/construction expressed by the left side is identical with the outcome of *the procedure*/construction expressed by the left side.

Thus what is the *sense*, the meaning of the left side expression and the meaning of the right side expression? The former is the procedure (construction) expressed by '3 + 5', the latter is the procedure (construction) expressed by '+ √16'. Indeed, these procedures are different, so the senses are different, the denotation (here 8) is the same.

The TIL analysis of the expression '3 + 5' results in the construction $[{}^0\!+\ {}^0 3\ {}^0 5]$. Every object 'contained' in a construction is represented by a construction. Thus + [the function], 3 and 5 (the numbers) are represented by 'trivializations': where X is any object (including constructions) the construction ${}^0X$ is a *trivialization*: it *mentions* X. In this way the object under a trivialization is constructed without any change. Our record of the construction expressed by '3 + 5' is therefore a kind of instruction (extra-linguistic):

1. Identify the function +;
2. Identify numbers 3, 5;
3. Apply the function + to <3, 5>.

A similar interpretation can be articulated as concerns '+ √16'.

These examples are extremely simple, of course. The procedural semantics used by TIL and based on the notion of construction is however universally applicable even in rather complicated cases (*de re, de dicto,* attitudes, anaphora, donkey sentences, tenses etc.). We will therefore say something more about TIL constructions (without definitions[5]) in order to be able to explain in more details the TIL triangle + reference.[6]

[5] These can be found in TIL literature, in particular in Tichý (1988) or Duží – Jespersen – Materna (forthcoming).

[6] The following survey is only fragmentary information serving to give a 'general impression' only.

TIL has been inspired by (typed) λ-calculi. It was Church's ingenious idea that abstract procedures can be essentially reduced to two of them: *creating functions* and *applying functions to arguments.* The TIL construction that creates functions is called *closure* (corresponding formally to *abstraction*), the construction that formally corresponds to *application* is called *composition*. The schema of the former is $\lambda x_1 \ldots x_m$ X, where $x_1, \ldots, x_m$ are pairwise distinct *variables* and X is a construction, the schema of the latter is $[\mathrm{X}\mathrm{X}_1 \ldots \mathrm{X}_m]$, where X, $\mathrm{X}_1, \ldots, \mathrm{X}_m$ are constructions. Constructions work in a type-theoretically classified setting and the objects are inputted *via variables* or *trivializations* (see above). Some further constructions can be added (like *double execution,* ^{2}X, defined if X is a construction that constructs a construction Y which constructs an object O: then ^{2}X constructs O). We will assume that *variables, trivialization, closure* and *composition* are at our disposal.

Since every construction is an *extra-linguistic* procedure, we must view variables as abstract procedures rather than as letters: they (as well as constructions that contain (free) variables) *v*-construct objects, where *v* is a parameter of *valuation*. The letters used for fixing particular variables (like *x, y, …, p, q, …, k, m, …*) are just names of variables. For each of the infinite number of types there is countably infinite number of variables at our disposal.

As for *types,* they are defined within a ramified hierarchy. Their choice is language dependent: analyzing natural languages in general we need another base of types than when we analyze some specific language like, e.g., a language of physics. Here we will consider just the types for analyzing natural language in general:

Atomic types of order 1:

o – truth-values (**T**, **F**);
ι – individuals;
τ – time moments/real numbers;
ω – possible worlds.

Functional types (of any order):

$(\alpha\beta_1 \ldots \beta_m)$ – the set of all partial functions from $\beta_1 \times \ldots \times \beta_m$ to α, where $\alpha, \beta_1, \ldots, \beta_m$ are types of the given order.

Ramified hierarchy:

The idea consists in defining first *constructions of order n* (they construct objects of types of order *n*) and then stipulating: $*_n$ be the set of constructions of order *n*. Then $*_n$ and the types of order *n* are *types of order n*+1.

Ramified hierarchy is of key importance: it makes it possible to not only *use* but also *mention* constructions (i.e., potential meanings) so that we do not need to use meta-language when speaking about constructions.

To make the previous 'technical' text more reader-friendly we will now give some simple *examples* of constructions and show how they can serve as meanings.

1. Consider the expression

(I) *(real) numbers greater than zero.*

Let us propose the construction which is the meaning of this expression. A *type-theoretical* analysis has to precede the analysis proper: The whole expression denotes a class of (real) numbers. Since TIL is based on functions rather than classes (and relations) every class is represented by its characteristic function. Here we get the type ($o\tau$). (This is the type of the object that is constructed by the construction which we have to find and which is the meaning of our expression.)

This function has to be created, so we must use a closure. Let *x* be a variable that *v*-constructs (= that ranges over) real numbers (we write $x \to \tau$). Our closure will be schematically

λx X,

where X is a construction *v*-constructing a truth-value. We can see that what X 'says' is

x is greater than zero.

So we have to ascribe types to *greater than* (>) and *zero* (0). The former is clearly ($o\tau\tau$) (see *Functional types* above) and *zero* can be considered to be a number, so $0/\tau$. We have to apply the (characteristic) function > to the pair $\langle x, 0\rangle$, so we use composition:

$[{}^0{>}\ x\ {}^0 0]$.

Now the whole construction is

$\lambda x\ [{}^0\!> x\ {}^0 0]$.

This construction (i.e., the procedure encoded by this sequence of characters) constructs (the characteristic function of) the class of positive real numbers.

2. The expression

(II) *mountains higher than Mont Blanc*

seems to be syntactically analogous to the preceding expression. But wait, this time we have to take into account the maybe inconspicuous but important fact that this expression is an *empirical* expression. Unlike the preceding example this expression does not denote a class: a modal and temporal variability is present. The mountains that are now higher than Mont Blanc are not *necessarily* higher, which can be stated by saying that there are other possible worlds where the same mountains are now not higher (while other mountains are) – this is modal variability. As for temporal variability, even in the actual world it does not hold that the mountains higher than Mont Blanc now were always and will always be higher. Thus what is denoted by our second example is not a class but a *property* (of individuals). The type of properties of individuals is $(((o\iota)\tau)\omega)$, abbreviated $(o\iota)_{\tau\omega}$. (So: functions that associate every possible world with a chronology of classes of individuals.)

So we have to find a construction such that it constructs a property of individuals, a property such as be possessed in any world-time by any individual that is a mountain *and* is greater than Mont Blanc. Types: M(ountain)/$(o\iota)_{\tau\omega}$, H(igher than)[7]/$(o\iota\iota)_{\tau\omega}$, Mont(Blanc)/$\iota$, $\wedge$/(ooo). Further the variables $w \rightarrow \omega$, $t \rightarrow \tau$, $x \rightarrow \iota$. We will create (discover) the searched construction in successive steps (as above).

First, to construct a function from possible worlds we get the form

λw X,

where X *v*-constructs a chronology: X $\rightarrow ((o\iota)\tau)$, so

7 Also an empirical expression, denoting an intension (the type is analogous to the type of M).

λt Y,

where Y *v*-constructs a class of individuals: Y $\rightarrow$ (οι), so

λx Z,

where Z *v*-constructs a truth-value: Z $\rightarrow$ ο. Now Z is obviously given by a conjunction. We have

$[^0\wedge\ [[[[^0\mathrm{M}w]t]x]\ [[[[^0\mathrm{H}w]t]x]\ ^0\mathrm{Mont}]]]$,

which can be in an obvious way abbreviated as follows:

$[^0\wedge\ [^0\mathrm{M}_{wt}x][^0\mathrm{H}_{wt}x\,^0\mathrm{Mont}]]$.

Omitting needless brackets we get the whole resulting construction (giving together the particular steps)

$\lambda w\ \lambda t\ \lambda x\ [^0\wedge\ [^0\mathrm{M}_{wt}x][^0\mathrm{H}_{wt}x\,^0\mathrm{Mont}]]$.

We can see that the 'syntactic' similarity between (I) and (II) is not as strong as it seems, in virtue of the ('inconspicuous') fact that (II), unlike (I), is an *empirical* expression.

It might seem that detecting the distinction we as if were too punctilious. Actually such distinctions, which in general distinguish logical form of empirical expressions from that of mathematical expressions, are connected with logically important consequences. The method of 'explicit intenzionalization'[8] as applied in our example makes it possible, e.g., to build up a highly expressive analysis of the logically surely relevant relation between *de re* and *de dicto* supposition (see Duží – Jespersen – Materna, forthcoming).

5 Empirical and Non-Empirical Expressions

Let us consider three kinds of non-indexical expressions of a natural language **L**.

a) Expressions containing exclusively logical[9] and mathematical subexpressions.

[8] See Jespersen (2005).

[9] It means 'used by two-valued partial logic'.

b) Non-empirical expressions containing at least one empirical subexpression.
c) Empirical expressions.

As for denotation/reference there is no problem with the category a). Here we can formulate no rational criterion which would make it possible to distinguish between denotation and reference. So let us use the term *denotation*. Clearly, any expression sub a) unambiguously denotes some abstract object or nothing at all. This concerns, e.g., logically true as well as only analytically true sentences: the logically true sentence

Three is greater than two or three is not greater than two.

as well as the analytically true sentence

Three is greater than two.

both *denote* the truth-value **T**. A case where the denotation is missing:

the greatest real number.

Here the class of 'greatest real numbers' is empty, and since *the* is the function that returns the only member of a singleton[10] and is undefined on other classes (i.e., those that are empty or contain more than one member) our expression cannot denote anything.[11]

The expressions that are typical representatives of the kind b) are analytically true or analytically false sentences. Due to the fact that they contain some empirical subexpressions the respective analysis must take it into account and result in a construction of the form

$\lambda w\, \lambda t\, \mathrm{X}$,

with X any construction, $\mathrm{X} \rightarrow \mathrm{o}$. To adduce a classical example consider the sentence

Every bachelor is a man.

[10] A singleton is a class that contains just one member.

[11] We have empty classes but we do not have 'empty individuals' or 'empty numbers'.

(The respective construction would be (Every, $\forall/(o(o\iota))$, $\supset/(ooo)$, Bach, $\text{Man}/(o\iota)_{\tau\omega}$):

$$\lambda w \lambda t\ [{}^0\forall\ \lambda x\ [{}^0\supset [{}^0\text{Bach}_{wt}x]\ [{}^0\text{Man}_{wt}x]]]\ )$$

The empirical subexpressions *bachelor, man* express constructions that construct *non-trivial intensions*, i.e., intensions that are not constant functions. He who understands the language **L** (here English) knows however that the whole expression expresses a construction that constructs a *trivial intension* (here a trivial *proposition*) whose value is the same in all worlds-times (here **T**).

Now since the meanings of the expressions of the category b) construct (trivial) intensions and some subexpressions express even constructions of non-trivial intensions a clear criterion of being the (a) reference of an expression can be articulated:

> **A** reference in the world W at the time T of an expression E that contains some empirical subexpression is the value of the denotation of E in W at T. **The** reference of an expression E is the reference of E in the actual world-time.

So the members of the category b) *denote* trivial intensions and their *reference* is the same in all possible worlds-times.

The category c) is also unambiguous: empirical expressions *denote* non-trivial intensions and their *references* are distinct in at least two possible worlds-times.

6 Necessity and Contingence in the Semantic Triangle

Summing up, assuming, as we do, that solving problems of LANL we accept the results of linguistic convention as being at our disposal (in other words, that we understand the expressions of the given language) we can state that

i) the link connecting an *expression* with its *meaning* is *a priori* in that its realization does not presuppose any extra-linguistic fact;
ii) the link connecting the *meaning* of the expression E with the *denotation* of E (if any) is *a priori* as well on the condition that the meaning is an abstract procedure and the denotation (if any) is the outcome of this procedure;

iii) the link connecting an *expression* E with the *denotation* (if any) is *a priori* because of i) and ii);
iv) the link connecting an *empirical expression* E with the *reference* of E, as well as the link connecting the *meaning* of the expression E with the *reference* of E are both *a posteriori*, i.e., the reference (if any) of an empirical expression cannot be determined by LANL itself: we need empirical steps.[12]

Remark: It was more than 10 years ago when the prominent Slovak philosopher Pavel Cmorej published a clear analysis of the distinction between *denotation* and *reference* (see his 1998). In his article an important point is emphasized: an explanation of the undeniable fact that the common intuition connects empirical expression with what we define as reference. He shows some situations where even the 'common people' not knowing semantic theories must admit that the way to the reference consists in complementing the link *expression – meaning – denotation* by empirical steps. Furthermore, some very interesting thoughts concerning the pragmatic relation *talk about* (including Donnellan's problem) can be found here and in (2000).

7 Some Consequences

A. *Emptiness*

In which case will we say that an expression

i) denotes nothing,
ii) refers to nothing?

Consider just the expressions not containing indexicals: the constructions that are meanings of such expressions are *closed* constructions, i.e., they do not contain any free variable. Closed constructions[13] have been shown to be good explications of what we mean by *concept* (see,

[12] The simple reason thereof is that no logical analysis can ever determine which of the possible worlds the actual one is. To know this is the same as to know all facts, i.e., to be omniscient.

[13] *Modulo* α- and η-equivalence (as defined in λ-calculi).

e.g., Materna 2009). So we can ask: When the concept expressed by an expression is an *empty concept?*

Case a): the concept is *strictly empty,* i.e., it does not construct any object.

This is the case when the concept is a composition (see **4**.) and the function constructed by X in $[XX_1\ldots X_m]$ is undefined on its arguments.

Examples: *the greatest prime, the result of 5:0…*

Only mathematical expressions come under this case. They do not denote anything (but we understand them because they possess meaning, of course).

Case b): the concept is *quasi-empty*, i.e., it constructs an empty class/relation.

Examples: *the even primes greater than two, being smaller than and identical to…*

This case also embraces just mathematical expressions. Thus no distinction between denotation and reference can be stated.

Case c): Empirical expressions: an empirical expression is *empirically empty* iff the respective concept constructs an intension whose value in the actual world-time is either an empty class/relation or is missing.

Examples: Russell's *the King of France, Pegasus, Unicorns, Brontosaurus…*

So they denote intensions and *the* reference of them does not exist or is an empty class/relation, while there can be *a* reference and a non-empty class as a reference in some possible worlds-times.

Case d): Non-empirical expressions containing empirical subexpressions: such an expression is, say, *analytically empty*[14] iff the respective concept constructs an intension whose value is in all worlds-times the same, viz. none or empty class/relation.

[14] No official terminology capturing the terms of this paragraph exists as yet.

Examples: *the oldest married bachelor, married bachelors…*

The reference is missing in all worlds-times (the first example) or an empty class in all worlds-times (the second example).

The reason why no expression containing empirical subexpressions can be strictly empty is simple: such expressions express concepts that construct intensions (trivial or non-trivial). Intensions are functions and any function is an object, even if the function is undefined on every argument.

All in all, with the only exception of some mathematical expressions (case a) above) every expression denotes something. Saying that an empirical expression is empty we claim that it does not *refer* to anything. The reference can be missing, never the denotation.

B *Empirical definite descriptions*

We know that some expressions that look like predicates, i.e. like denoting classes/properties, but actually concern particulars: consider the expressions

the Pope, the King of France, the highest mountain…

Whoever understands these expressions knows that there cannot be more than one Pope, one King of France, one highest mountain. By contrast, the genuine predicates are indifferent to the cardinality of their contingent 'population'. Russell in his (1905) attempted to capture this difference in terms of combining a predicate with the descriptor *the* (Russell's *iota inversum*). His well-known attempt at eliminating description itself as lacking any self-contained semantics was obviously motivated by the effort not to admit truth-gaps. The positive feature of Russell's idea (as compared with Frege) was that the (empirical) description *the F* does *not* refer to a particular individual: this discovery was necessary to avoid troubles with non-referring expressions like *the King of France.* On the other hand the claim that the description itself (beyond the context of predication) does not mean anything is strongly counterintuitive (do we not understand the expressions *the Pope, the King of France, the highest mountain* etc.?).

Russell's elimination can be and has been criticized, and Strawson's (1950) saved our intuition that truth-gaps are sometimes unavoidable. Here we want just to exploit our conception of distinguishing between denotation and reference.

First of all, Russell's *the* is for us (in contrast to Russell) not an 'improper symbol'. Its type is type-theoretically polymorph, so let it be schematically $(\alpha(o\alpha))$ for any type α. *The* is thus a partial function that behaves as follows: if applied to a class whose unique member is an object K it returns K as its value. In the other cases, i.e., if the class is empty or contains more than one member, the function *the* is undefined, returns nothing. So what can be said about the expression *the King of France*?

As an empirical expression it surely does denote something, viz. an intension called *individual role*, type $\iota_{\tau\omega}$. The reference is of course missing. In this case the whole construction that is the meaning of the left side of Russell's elimination equation constructs a proposition that is truth-less in those worlds-times where there is no King of France, while the meaning of the right side constructs a proposition that possesses a truth-value even in the worlds where there is no King of France. So we see that the equation does not hold as soon as we admit truth-gaps. (Strawson was right.)

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Filosofický ústav AV ČR
Jilská 1
110 00 Praha 1
MaternaPavel@seznam.cz

References

CHURCH, A. (1956): *Introduction to Mathematical Logic*. Princeton: Princeton University Press.

CMOREJ, P. (1998): Denotácia a referencia. In: Cmorej, P. (ed.): *K filozofii jazyka, vedy a iným problémom*. Príloha časopisu *Organon F*. Bratislava: Infopress, 7 – 19. (Reprinted with some adjustments in Cmorej 2001.)

CMOREJ, P. (2000): Referencia, identifikácia a intencionálne akty. In: Cmorej, P. (ed.): *Denotácia, referencia a význam*. Príloha časopisu *Organon F*. Bratislava: Filozofický ústav SAV, 29 – 45. (Reprinted in Cmorej 2001.)

CMOREJ, P. (2001): *Na pomedzí logiky a filozofie.* Bratislava: Veda.

CRESSWELL, M.J. (1975): Hyperintensional Logic. *Studia Logica*, Vol. 34, 25 – 38.

CRESSWELL, M.J. (1985): *Structured Meanings*. Cambridge: MIT Press.

DUŽÍ, M. – JESPERSEN, B. – MATERNA, P. (forthcoming): *Procedural Semantics for Hyperintensional Logic*. Springer 2010.

FREGE, G. (1892): Über Sinn und Bedeutung. *Zeitschrift für Philosophie und philosophische Kritik* 100, 25 – 50.

FREGE, G. (1971): *Schriften zur Logik und Sprachphilosophie.* (*Aus dem Nachlass).* G. Gabriel (ed.). Hamburg: Felix Meiner Verlag.

GEACH, P. – BLACK, M. (eds.) (1952): *Translations from the Philosophical Writings of Gottlob Frege.* Oxford: Basil Blackwell.

JESPERSEN, B. (2003): Why the Tuple Theory of Structured Propositions Isn't a Theory of Structured Propositions. *Philosophia* 31, 171 – 83.

JESPERSEN, B. (2005): Explicit Intensionalisation, Anti-Actualism, and How Smith's Murderer Might Not Have Murdered Smith. *Dialectica* 59, 285 – 314.

MATERNA, P. (2009): Concepts and Recipes. *Acta Analytica* 24, 69 – 90.

RUSSELL, B. (1905): On Denoting. *Mind* 14, 479 – 93.

STRAWSON, P.F. (1950): On Referring. *Mind* 59, 320 – 44.

TICHÝ, P. (1968): Smysl a procedura. *Filosofický časopis* 16, 222 – 232. Translated as 'Sense and Procedure' in: Tichý (2004), 77 – 92.

TICHÝ, P. (1969): Intensions in Terms of Turing Machines. *Studia Logica* 26, 7 – 25. Reprinted in: Tichý (2004), 93 – 109.

TICHÝ, P. (1988): *The Foundations of Frege's Logic.* Berlin, New York: De Gruyter.

TICHÝ, P. (1994): The Analysis of Natural Language. Reprinted in: Tichý (2004), 801 – 41.

TICHÝ, P. (2004): *Collected Papers in Logic and Philosophy*. V. Svoboda, B. Jespersen, C. Cheyne (eds.). Prague: Filosofia, Czech Academy of Sciences, and Dunedin: University of Otago Press.

ZALTA, E. (1988): *Intensional Logic and the Metaphysics of Intentionality*. Cambridge, London: MIT Press.