POINTS OF VIEW FROM A LOGICAL PERSPECTIVE (II)

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ABSTRACT. In the paper we offer a *logical explication* of the frequently used, but rather vague, notion of *point of view*. We show that the concept of point of view prevents certain paradoxes from arising. A point of view is a means of *partial* characterisation of something. Thus nothing is a *P* and at the same time a non-*P* (*simpliciter*), because it is a *P* only relative to some point of view and a non-*P* from another point of view. But there is a major, complicating factor involved in applying a logical method that is supposed to provide a formal and rigorous counterpart of the intuitively understood notion: 'point of view' is a homonymous expression, and so there is not just one meaning that would explain points of view. Yet we propose a common scheme of the logical type of the entities denoted by the term 'point of view'. It is an empirical *function*: when applied to the viewed object in question, it results in a (set of) evaluating proposition(s) about the object. If there is an agent applying the criterion, the result is the agent's *attitude* to the respective object. The paper is organised into two parts. In Part I we first adduce and analyse various examples of typical cases of applying a point of view to prevent paradox. These cases are examined according to the type of the viewed object: a) the viewed object is an individual and b) the viewed object is a property or an office. In Part II we then show that the method described in Part I can be applied also to the analyses of agents' attitudes. We explain how an agent can believe of something that it is a *P* and at the same time a non-*P*: the agent applies different viewpoint criteria to the viewed object. The inversion of perspective consisting in the perspective shifting from the believer on to the reporter in the case of attitudes *de re*, and from the reporter to the believer in the case of attitudes *de dicto*, is also analyzed. We show that there is no smooth logical traffic back and forth between such attitudes unless some additional assumptions are added, and prove that they are not equivalent. By way of conclusion, we explicate the notion of *conceptual* point of view and analyze cases of viewpoints given by conceptual distinction. We show, finally, that the proposed scheme of the type of point of view can be preserved, this time, however, in its extensional version.

KEYWORDS. Point of view, perspective, transparent intensional logic, intension, requisite, attitudes, *de dicto/de re* supposition, conceptual viewpoint, definition.

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4. Points of view, attitudes and inversion of perspective

In Part I we introduced a common scheme of the logical type of the entities denoted by the term 'point of view'. It is an empirical *function*: when applied to the respective criterion of evaluation and an abstract or concrete object it results in an evaluating proposition about the object: $V(iew)/(o_{\tau\omega}\ \beta\ \alpha)_{\tau\omega}$. As above, α, β are any types, β the type of the 'viewing criterion', α the type of the object that is viewed by an agent. We analysed typical cases of applying a point of view to prevent paradox. These cases were examined according to the type of the viewed object: a) the viewed object is an individual and b) the viewed object is a property or an office. Now we are going to show that the method described in Part I can be applied also to the analyses of agents' attitudes.

It might seem that if the agent applies a point of view to an object, i.e., if he (correctly) evaluates the criterion and obtains the resulting proposition, then he has to assent to the statement claiming the truth of the obtained proposition *P*. In other words, the agent's *believing* that *P* should be justified, and the association between the respective point of view and an attitude to *P* would be the relation of logical consequence or otherwise logical in nature. For instance, if somebody applies the 'age-criterion' to 80-year old Charles, obtaining thus the proposition that Charles is an old man, he should believe (and know) that Charles is an old man. However, we will show that situation is not that simple. For it is logically possible that an agent applies a criterion to an object, yet fails to believe that the resulting proposition *P* is true, if the proposition is presented to him in an incomprehensible way. Hence the agent may fail to adopt the attitude that *P*. We envisage interplay between agents, points of view and attitudes consisting in an agent's applying a point of view, and arriving at an attitude. The idea behind, as it were, slotting points of view in between agents and attitudes is to provide a rationale for an agent to adopt an attitude. So what follows below is an outline of attitude acquisition *via* points of view *per se*.

In general, what we have called '*viewing criterion*' is always some *attribute* or a set of attributes, whose application to the viewed object or—if the viewed object is an intension—to the bearers of an intension determines the resulting proposition. A possible dialogue could elucidate the theoretical analyses above.

A: Yesterday I met Charles. He is already rather old.
B: Nonsense! Charles is in many respects younger than many of us.
A: What do you say! He is already over 70.
B: Well, from the viewpoint of age you are justified in claiming that he is old. I only say that the viewpoint of age is not the only possibility.
A: What do you mean? I don't understand.
B: Did you notice his activities? He plays tennis, swims rather quickly, organises some linguistic courses, is full of various interests, so I would really say that he is rather young.
A: Why not, but then you believe that he is old and, at the same time, young. A bit strange, wouldn't you say?
B: I see that your language game is 'dialectic'. I do not accept the paradoxical character of your claim; to be old from the viewpoint of age is well compatible with being young from the viewpoint of activities, these two predicates are not antonyms.

***Remark*:** Viewpoints concern in most cases *vague* predicates. Applying a viewpoint can be construed as a kind of explication. (Remember Old^{Age}, Young^{Act}.) The fact that such 'viewpoint-like' explications are not 'absolute' and that they are dependent on the agent who applies *his/her* viewpoint can be seen from the very scheme of the viewpoint function:

[$View_{wt}$ *Criterion Object*].

Let a particular criterion be fixed (say, *Age*) as well as the object (say, *Charles*). Consider a definite value of the criterion applied to the object (say, 60 years). The proposition produced by the particular viewpoint may be *Charles is* old^{Age} but it may be also *Charles is not* old^{Age}. This means that the same argument is as if connected with two values, which would show that the *View* would not be a function in spite of its type. This is impossible, of course. So we have to admit that the *View*s may be distinct for the same argument. Here we can see that this theoretical possibility is frequently realised in practice due to the fact that the various distinct *View*s are selected by various agents whose ways of evaluating the criterion may differ.

It would seem that we could set up a theory of *attitudes* by defining relations-in-intension between agents, of type ι, and (the results of points of view, i.e.) propositions, of type $o_{\tau\omega}$. In §2 the type of propositional attitudes was already indicated, to wit $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$.

The element of attitude logic we will be concerned with here concerns the perspective that each attitude comes with. It is a thrice told tale that $a < b$ and $b > a$ determine one and the same situation (whether the situation is a mathematical fact or an empirical state-of-affairs), involving two distinct perspectives on the same situation. We have to teach a student to adopt a correct logical perspective on 'it is not true that if *A* then *B*' so that she would know that '*A* and not *B*' is equivalent to the former, denoting the same truth-condition. Otherwise she may easily believe the former without believing the latter. The notion of difference in perspective may be captured in various ways. One way, in particular, is to distinguish between attitudes *de dicto* and attitudes *de re*. Another, maybe more promising, way is to distinguish between attitudes to the 'situation', i.e., to a proposition, and attitudes to a *construction* of this proposition, the *way* in which the proposition is presented. Therefore, besides 'propositional attitudes' of type $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$ we will also investigate constructional 'hyper-propositional attitudes' of type $(o\ \iota\ *_n)_{\tau\omega}$. We will call the former *implicit* attitudes and the latter *explicit* attitudes. The distinction between implicit and explicit attitudes enables us to distinguish between 'rational and irrational' behaviour of an agent. In the case of *implicit* attitudes to a proposition the agent cannot behave irrationally. If he has applied a particular viewpoint and evaluated the criterion leading to a proposition *P*, then it logically follows that the agent (implicitly but *not explicitly*) accepts *P* and has an attitude (of justified belief) to *P*. First, we will investigate the *de dicto* and *de re* forms of implicit attitudes, and at the end of this section we examine *de dicto* and *de re* explicit attitudes. We will analyse sentences of the form

X B's that the *F* is a *G*

(e.g., Charles believes that the Pope is in danger)

to be contrasted with

The *F* is *B*(ed) by *X* to be a *G*

(e.g., The Pope is believed by Charles to be in danger)

where *B* is an attitude verb like 'believe', 'know', 'doubt', 'think', etc., *X* is an *agent* (*alias* '*believer*'). Since there is also somebody who reports on the situation, and who matters here as well, we will call this person a *reporter*.[1]

[1] A sophisticated distinction between *reporting* and *attributing* an attitude is irrelevant to our purposes here.

What we propose here is to juxtapose attitudes *de dicto* and their *de re* counterparts. Our main reason for so doing is that an *inversion of perspective* comes to the fore. In general, using a *de dicto* form, the reporter is supposed to loyally represent the perspective that the agent has on some particular matter. On the other hand, using a *de re* form, the reporter is responsible for ascribing the property of being believed by the agent to be the/an F.[2] Consider, for instance, the following two attributions:

(1) *Charles believes that the Pope is in danger*.
(the Pope occurs *de dicto*)

(2) *Charles believes of the Pope that he is in danger*.
(the Pope occurs *de re*)

Somebody might wonder though, whether there is any semantic difference between (1) and (2). An example may serve to bring out the need to distinguish between attitudes *de dicto* and *de re*. Consider a variant of (1) and (2):

(3) *Charles believes that the Pope is not the Pope*.

(4) *Charles believes of the Pope that he is not the Pope*.

In (3) we balk at attributing to Charles the belief that the Pope is not the Pope, if by this is meant that Charles believes that (the person who is) the Pope is not the Pope. Only "paraconsistent popes" are capable of pulling this trick. But there is a way around attributing a paradoxical attitude to Charles, namely by having him believe of the person who is the Pope that he is not the Pope. An equivalent formulation is that the person who is the Pope is such that he is believed by Charles not to be the Pope:

(5) *The Pope is believed by Charles not to be the Pope*.

It is the reporter who is responsible for creating an air of paradox by employing the individual office of the Pope twice over, for any office co-occupied by the Pope would have served equally well. E.g., the reporter might instead have said that the German with the highest clerical rank in the Vatican is such that he is believed by Charles not to be the Pope; equivalently, that Charles believes of the German with the highest clerical rank in the Vatican that he is not the Pope. In general, the property of being believed by Charles to be an or the Φ is type-theoretically no dif-

[2] '*X* believes of the *F* that it is a *G*' and 'The *F* is believed by *X* to be a *G*' are treated as equivalent formulations.

ferent from the property of being happy, say. Its construction, on the other hand, is somewhat more involved than the construction of *being happy*. The point is now that if the offices of the Pope and of the German with the highest clerical rank in the Vatican are co-occupied at the world and time where Charles has an attitude *de re* involving one of the offices, then the other office may be substituted for it. The reporter uses the office only as a 'pointer' to the individual to whom he ascribes the property of being believed by Charles not to be the Pope.

This is evidently not the case with respect to attitudes *de dicto*. Our method of distinguishing between attitudes *de dicto* and *de re* turns on whether the relevant office is picked out in a *de dicto* or *de re* way. In the former case the construction of the office is used *de dicto* and the whole office is thus the object of predication. In the latter case the construction of the office is used *de re* and the occupant of the office at the given world and time is the object of predication.

Here is a schematic example of how an attitude *de dicto* and an attitude *de re* are constructed.[3] For the sake of simplicity, we investigate first *implicit* attitudes *B* of type $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$. Moreover, we have here Ch/ι, $F/\iota_{\tau\omega}$, $G/(o\iota)_{\tau\omega}$, x ranging over ι:

(6) $\lambda w\lambda t\ [{}^0B_{wt}\ {}^0Ch\ \lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ {}^0F_{w^*t^*}]]$

(${}^0F$ *de dicto* – '… that the *F* is *G*')

Here the whole proposition that the *F* is a *G* (constructed by $\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ {}^0F_{w^*t^*}]$) is the object of predication, and so is the whole office *F*.

(7) $\lambda w\lambda t\ [\lambda w\lambda t\ \lambda x\ [{}^0B_{wt}\ {}^0Ch\ \lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ x]]]_{wt}\ {}^0F_{wt}]$,

or equivalently

(7β) $\lambda w\lambda t\ [\lambda x\ [{}^0B_{wt}\ {}^0Ch\ \lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ x]]\ {}^0F_{wt}]$

(${}^0F$ *de re* – '… of the F that it is G')

Analysis (7) may call for elucidation. The *de re* case can be read as:[4]

The F is (i.e., has a property of being) believed by Charles to be a G.

The property of being believed by Charles to be a G is constructed as follows:

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[3] For technical explanations see Duží (2004).

[4] Here we analyse the passive variant like (5) of the *de re* attitude. For a direct analysis of the active variant by using *Sub* and *Tr* functions, see (9ββ) below or, e.g., Duží (2006).

$[\lambda w \lambda t\ \lambda x\ [{}^0B_{wt}\ {}^0Ch\ \lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ x]]]$

Applying this property at a given w,t to the occupant of the office F (at that w,t), we obtain the desired analysis (7). The construction of the office F has been subjected to intensional descent with respect to reporter's perspective (i.e. composed with reporter's w, t).

It is a complicating factor that a construction of an office which has already undergone intensional descent may still occur *de dicto* in a larger context, particularly in the case of attitudes *de dicto*. The philosophical motivation is that the existential requirement associated with an occurrence *de re* must be suspended in certain cases. In our examples (1) and (2), F is the office of the Pope, G is the property of being in danger. Thus whereas (1) may still be true even if there is no Pope, (2) will have *no truth-value* in such a case. The truth-condition, or proposition, constructed by (6) is fulfilled by any world-time pair WT where Charles has a belief to the effect that the *whole proposition is true* (regardless whether there is a unique F there and then; the proposition, and thus the office F, is *mentioned*). The asterisks are a heuristic device fixing which occurrences of w, t fall inside Charles' perspective.

On the other hand, the truth-condition constructed by (7) is fulfilled by any WT where F at *this WT* is an element of the set which is (in *this WT*) the extension of the property of being believed by Charles to be a G. Notice that the indices of F are outside Charles' perspective this time. The reporter might just as well have used any other co-occupied (in W, T) office as a pointer to the individual that has the property of being believed by Charles to be a G. In other words, (7), as well as its negated form, namely

$\lambda w \lambda t\ [\lambda w \lambda t\ \lambda x\ \neg[{}^0B_{wt}\ {}^0Ch\ \lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ x]]]_{\mathrm{wt}}\ {}^0F_{wt}]$,

entail that the F exists in W, T.

Recalling our preliminary characterisation of the *de dicto*/*de re* distinction (see §2): Construction C is used in the *de re* supposition in a construction C' iff C is subjected in C' to an intensional descent with respect to reporter's perspective w, t, otherwise C occurs *de dicto* in C'. But what is the difference between those two kinds of attitudes after all? For, are those two constructions not β-equivalent? No, they are not, appearances notwithstanding. Even the asterisk notation 'renaming' variables w, t, which does prevent collision of variables, does not make it possible to equivalently perform the respective β-reduction, for we would get:

(7β^r) $\lambda w \lambda t\ [{}^0B_{wt}\ {}^0Ch\ \lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ {}^0F_{wt}]]$.

Now (7β^r) is obviously not equivalent to (6); ${}^0F$ has been subjected to intensional descent, this time with respect to the *reporter's* perspective, not Charles'. Hence the construction of the office *F* still occurs *de re*, not *de dicto* according to the above characteristics. Worse, (7β^r) is not even equivalent to (7β). Since *F* is a properly partial function, in some *WT* pairs the construction ${}^0F_{wt}$ is *v*-improper. Due to the Compositionality principle, the partiality of one constituent of a complex renders the entire complex vulnerable to partiality[5] (partiality is "propagated up"). Therefore, if ${}^0F_{wt}$ is *v*-improper at some *WT*, the whole composition

$[\lambda x\ [{}^0B_{wt}\ {}^0Ch\ \lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ x]]\ {}^0F_{wt}]$

is *v*-improper, and the proposition constructed by (7β) does not have a truth-value in such a *W* at such a *T*. On the other hand, λ-closure can never be *v*-improper.[6] For those *WT*-pairs where the construction ${}^0F_{wt}$ is *v*-improper, the closure $\lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ {}^0F_{wt}]$ *v*-constructs a degenerated function not defined on any argument, and the proposition constructed by (7β^r) *can* be true or false (Charles *can* be related to such a degenerated function). In other words, (7β^r) does not entail the existence of the *F*, unlike (7β) and (7).

Thus the correct analysis of the *de re* case is a non-reduced form (7) or (7β), but not (7β^r). Partiality, which models the *WT*-relative vacancy of offices, throws a spanner in the works by blocking unrestricted β-conversion between the constructions of attitudes *de re* and *de dicto*. The above characterisation of the *de dicto/de re* occurrence has thus to be adjusted as follows:

> Let $C' \to \alpha_{\tau\omega}$ occur as a constituent (i.e. used subconstruction) of a construction *C*. The occurrence of *C'* is used in the *de re* supposition in the construction *C* iff *C'* occurs in the composition with the constructions of its arguments, i.e. C'_{wt}, and *C'* does not occur as a constituent of another proper subconstruction *C''* of *C* (i.e., ${}^0C'' \neq {}^0C$) such that *C''* occurs in the *de dicto* supposition (the *de dicto* context is dominant). Otherwise *C'* occurs *de dicto* in *C*.[7]

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[5] See Part I, Definition 2, Note 3.

[6] See Part I, Definition 2, Note 4.

[7] In Duží (2004) the occurrence of ${}^0F$ in the problematic construction (7β^r) has been labelled as *de re*. In Duží (2006, 202 – 203) the question whether such an occurrence

The truth-condition of (7) comes with an additional requirement which is not found in (6). The additional constraint is that *F* must be non-vacant at *WT*. And whether it is, is no trivial matter since *F* is a properly partial function. E.g., some world-time pairs have no Pope to offer at all, others have exactly one. Whether there is a Pope or not, Charles may well believe that the Pope is happy. Analogously, whether or not the site of Troy exists, Schliemann may still seek the site of Troy. But Schliemann could not possibly find the site of Troy, if there was no site of Troy to be found in the first place. By the same token, it is impossible for Charles to believe of the Pope that he is happy, if there is no Pope around of whom to believe that he is happy. This is parallel to claiming that if there is no unique Pope at *WT* then it is neither true nor false that the Pope is happy at *WT*. The bottom line is that attitudes *de re* come with an existential presupposition, and it is interesting to note that this holds whether these attitudes are taken to be 'difficult' or 'easy' to come by. In either case the existence of some particular *res* is required.

What attitudes *de re* are supposed to do, on the other hand, is a somewhat contentious issue. R. Foley distinguishes between two accounts of beliefs *de re*:

> [T]hose accounts that make it relatively difficult to believe *de re* of an object that it has some characteristic, because they require believers to have a special, intimate relation of some sort with objects about which they have *de re* beliefs; and those accounts that do not require there to be such a relation and thus make it relatively easy to have *de re* beliefs (Foley 1986, 332 – 333).

The difference is whether the agent either must or need not know which object is the *res* in question. This rather vague formulation can be made more precise using TIL to distinguish between offices and their occupants. The agent, or believer, must have a kind of an 'intimate relation' to the individual that is singled out by the reporter *via* an office. But the agent does not have to connect this individual with the respective office. The reporter uses the office as a pointer to the *res*, the believer does not. Hence our account is of Foley's second kind. Indeed, the agent of an attitude *de re* need do absolutely nothing to entertain such an attitude. What he must do, however, is having already adopted some other atti-

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should be called *de dicto* or *de re* has been left open. However, in both papers we warned against an inadequate use of the β-reduced construction of type ($7\beta^r$) for analysing sentences with the *de re existential presupposition.*

tude, whether *de dicto* or *de re*, since attitudes *de re* on our construal are parasitic on prior attitudes.

The inversion of perspective alluded to above consists in the perspective shifting from the believer on to the reporter in the case of attitudes *de re*, and from the reporter to the believer in the case of attitudes *de dicto*.[8] One crucial upshot of handling these notions in TIL, however, is that there is no smooth logical traffic back and forth between such attitudes. They are not equivalent. But the inversions will begin to flow once an additional premise is added, providing both the fulfilment of the existential requirement and the agent's knowledge of who satisfies the requirement. In this connection it is important to underline that it makes no logical difference if the roles of reporter and believer are played by the same individual.

Our claim is that the proposition constructed by (6) does not entail the proposition constructed by (7), nor *vice versa*. To see this, consider the following. Proposition (6) does not entail proposition (7), because the latter has an existential presupposition that (6) does not have. In (6) the issues whether F is vacant and, if occupied, who happens to occupy F are irrelevant (from the reporter's point of view). Not so with (7), where the requirement crops up and it is relevant if some distinct office F' happens to share its occupant with F at WT. (7) does not entail (6): Even if F is occupied (and Charles has some 'intimate relation' to an individual who happens to be the F), it is not *eo ipso* so that Charles has any (*de dicto*) belief about F. And even if Charles does have any beliefs about F, and there is some F' such that F, F' coincide at WT, it is not *eo ipso* so that Charles has any (*de dicto*) beliefs about F'. Though the following argument

At WT, Charles believes that the F is a G
At WT, Charles knows that the F is the F'
$\therefore$ At WT, Charles believes that the F' is a G

is valid, it still does not enable us to logically transform the above *de dicto* beliefs to the *de re* ones, or *vice versa*. Charles' knowledge that the F is the F' entails that both offices are occupied at WT thanks to the factivity of knowledge:

At WT, Charles knows that the F is the F'.
Hence, at WT, the F is the F'.
Hence, at WT, F and F' are occupied.

[8] For inversion of perspective, see also Jespersen (2001).

Formally:

$$[{}^0K_{wt}\ {}^0Ch\ \lambda w^*\lambda t^*\ [{}^0F_{w^*t^*} = {}^0F'_{w^*t^*}]]$$
$$\therefore\ [{}^0F_{wt} = {}^0F'_{wt}]$$
$$\therefore\ [{}^0Occ_{wt}\ {}^0F],\ [{}^0Occ_{wt}\ {}^0F'].$$

K receives the same type as *B*, while *Occ*(upied) is an empirical property of offices, of type $(o\ \iota_{\tau\omega})_{\tau\omega}$.

The problem consists in the fact that Charles does not have to know *who is the occupant* of either office. Imagine a situation where Charles reads in a trustworthy journal that the Pope is the head of the Roman-Catholic Church and as such is in danger. Being an ignorant, our Charles does not know that Joseph Ratzinger is the Pope, and the reporter is not entitled to claim that Charles believes *of the Pope* that he is in danger. On the other hand, Charles may believe that Ratzinger is in danger without knowing that Ratzinger is the Pope. In such a situation the reporter may claim that Charles believes of the Pope that he is in danger, but cannot truly make a *de dicto* claim to the effect that Charles believes that the Pope is in danger.

To be able to mutually transform *de dicto* and *de re* attitudes, we need still a stronger premise, swapping knowledge for belief, namely:

Charles *knows* that the *F* is (occupied by) *X*. $(F/\iota_{\tau\omega},\ X/\iota)$

This fresh premise kills two birds with one stone. First, due to factivity of knowledge, it is true that the *F* is *X*:

$$\lambda w\lambda t\ [{}^0F_{wt} = {}^0X]$$

and

$$\lambda w\lambda t\ [{}^0Occ_{wt}\ {}^0F].$$

Second, Charles' knowledge of the (contingent) occupation of *F* at *WT* by *X* entails that *X* belongs to the extension of the property of being believed by Charles to be a *G*, and since *X* is the value at *w, t* of the intension constructed by ${}^0F$, the respective *de dicto* and *de re* attitudes are mutually transferable '*via X*'. However, in order to perform the transform, we need another assumption that knowing entails believing. This is plausible providing we consider the ordinary meaning of 'knowing' and 'believing'. Thus we dismiss situations like when somebody "knows that *P*" without being convinced of it. Thus the additional assumptions and their consequences are as follows:

(i) Charles knows that the *F* is (occupied by) *X*;
(i′) $\lambda w\lambda t\,[{}^0K_{wt}\,{}^0Ch\,[\lambda w\lambda t\,[{}^0F_{wt} = {}^0X]]]$.

Since knowing is a *factivum*, (i) entails

(ii) The *F* is (occupied by) *X*;
(ii′) $\lambda w\lambda t\,[{}^0F_{wt} = {}^0X]$.

Since knowing entails believing, (i) entails

(iii) Charles believes that the *F* is *X*;
(iii′) $\lambda w\lambda t\,[{}^0B_{wt}\,{}^0Ch\,[\lambda w\lambda t\,[{}^0F_{wt} = {}^0X]]]$.

The inversion of perspective then comes about in the following manner:

a) *De dicto* → *de re*

(iv) Charles believes that the *F* is a *G*;
(iv′) $\lambda w\lambda t\,[{}^0B_{wt}\,{}^0Ch\,[\lambda w\lambda t\,[{}^0G_{wt}\,{}^0F_{wt}\,]]]$.

By introducing a conjunction we get:

(v) Charles believes that the *F* is a *G* and also that the *F* is *X*;
(v′) $\lambda w\lambda t\,[[{}^0B_{wt}\,{}^0Ch\,[\lambda w\lambda t\,[{}^0G_{wt}\,{}^0F_{wt}\,]]] \wedge [{}^0B_{wt}\,{}^0Ch\,[\lambda w\lambda t\,[{}^0F_{wt} = {}^0X]]]]$.

Now, we consider the implicit belief, i.e., the relation of *Charles* to a proposition, an object of type $o_{\tau\omega}$. However, if *Charles* believes, knows, doubts, etc., a proposition *P*, then it does not mean that *Charles* is able to grasp the whole intension $P/o_{\tau\omega}$, i.e., the actual uncountable infinity. Nobody (except perhaps omniscient God) is able to do so. But when understanding the meaning $\lambda w\lambda t\,[{}^0G_{wt}\,{}^0F_{wt}\,]$, he is able to evaluate the instruction in *any W,* at *any T*; in other words, he has an access to the potential infinity. And in any *W* at any *T* when *Charles* evaluates the propositions constructed by $[\lambda w\lambda t\,[{}^0G_{wt}\,{}^0F_{wt}\,]]$ and $[\lambda w\lambda t\,[{}^0F_{wt} = {}^0X]]$ as being true, he also assents to the fact that $[[{}^0G_{WT}\,{}^0F_{WT}\,] \wedge [{}^0F_{WT} = {}^0X]]$ gives true, and also that $[{}^0G_{WT}\,{}^0X]$ is true. In other words, from the implicit-belief point of view the state of affairs *WT* in which both the propositions are true is indistinguishable from the state of affairs *WT* in which the proposition constructed by $[\lambda w\lambda t\,[{}^0G_{wt}\,{}^0X]]$ is true. Hence Charles is committed to believe the latter:

(vi) Charles believes that *X* is a *G*;
(vi′) $\lambda w\lambda t\,[{}^0B_{wt}\,{}^0Ch\,[\lambda w\lambda t\,[{}^0G_{wt}\,{}^0X]]]$.

By abstracting over *X* we get

(vii) *X* is believed by Charles to be a *G*;
(vii′) $\lambda w \lambda t\, [\lambda x\, [{}^0B_{wt}\, {}^0Ch\, [\lambda w \lambda t\, [{}^0G_{wt}\, x]]]\, {}^0X]$.

Now (ii) and (vii) give by substitution of identicals

(viii) The *F* is believed by Charles to be a *G*;
(viii′) $\lambda w \lambda t\, [\lambda x\, [{}^0B_{wt}\, {}^0Ch\, [\lambda w \lambda t\, [{}^0G_{wt}\, x]]]\, {}^0F_{wt}]$.

b) *De re* → *de dicto*

(ix) The *F* is believed by Charles to be a *G*;
(ix′) $\lambda w \lambda t\, [\lambda x\, [{}^0B_{wt}\, {}^0Ch\, [\lambda w \lambda t\, [{}^0G_{wt}\, x]]]\, {}^0F_{wt}]$.

Since the *F* is *X*, (ii) and (ix) give

(x) *X* is believed by Charles to be a *G*;
(x′) $\lambda w \lambda t\, [\lambda x\, [{}^0B_{wt}\, {}^0Ch\, [\lambda w \lambda t\, [{}^0G_{wt}\, x]]]\, {}^0X]$.

Since ^{0}X cannot be *v*-improper, by β-reduction (x) entails

(xi) Charles believes that *X* is a *G*;
(xi′) $\lambda w \lambda t\, [{}^0B_{wt}\, {}^0Ch\, [\lambda w \lambda t\, [{}^0G_{wt}\, {}^0X]]]$.

By introducing a conjunction to (iii) and (xi) we get:

(xii) Charles believes that *X* is a *G* and also that the *F* is *X*;
(xii′) $\lambda w \lambda t\, [[{}^0B_{wt}\, {}^0Ch\, [\lambda w \lambda t\, [{}^0G_{wt}\, {}^0X]]] \wedge [{}^0B_{wt}\, {}^0Ch\, [\lambda w \lambda t\, [{}^0F_{wt} = {}^0X]]]]$.

Now, by way of the same reasoning as above (xii) entails (xiii):

(xiii) Charles believes that the *F* is a *G*;
(xiii′) $\lambda w \lambda t\, [{}^0B_{wt}\, {}^0Ch\, [\lambda w \lambda t\, [{}^0G_{wt}\, {}^0F_{wt}]]]$.

The proof relies heavily on the fact that attitudes to propositions —implicit attitudes—are closed under the relation of logical entailment. Moreover, an essential assumption is Charles' knowing that the *F* is occupied by *X*. This *X* enables us to perform a both-ways transformation *via* β-conversion, because it cannot be subjected to intensional descent and thus be *v*-improper.

To sum up, a prerequisite for attributing an attitude *de re* to someone is that the reporter must know that the relevant office is occupied at *WT*, and if he wishes to substitute *F′* for *F* he must know that they coincide at *WT*. The believer need know neither. Substituting a construction of one (co-occupied) office for another in the case of attitudes that are already *de re* is one way of generating another attitude *de re*. A different way is to take as

premises an attitude *de dicto* involving *F* and the believer's knowledge of who is the *F* at *WT*, and deduce the conclusion that the believer has an attitude *de re* about *F*. An attitude *de re* can't come into being *ex nihilo*, because the agent needs first to believe or know that somebody is something in order to introduce a property and an office. Only then can the reporter begin to query whether the office is occupied so that the agent's attitude content (i.e., a construction) can be approached from the reporter's own vantage point. On this point we are in agreement with Foley:

> [C]ases involving *de re* beliefs about epistemically remote objects are cases where it is plausible to think that the person has these beliefs in virtue of having other beliefs… (Foley 1986, 341).

The ascription of any attitude is, of course, wrapped within the reporter's inescapably idiosyncratic perspective, but the attribution must be such that the believer not only would, but must endorse it as his own. This requirement is typically cashed out in the demand that the believer, if confronted with a sentence describing the attitude attributed to him, would, and rationally must, assent to it.

Yet, as stated above, an agent sometimes behaves as if irrational. An agent does apply a point of view, evaluates the respective criterion, obtaining thus a proposition, and still does not assent to this proposition. How is this possible? The problem consists in the fact that the agent's beliefs and knowledge are sensitive to the *way in which the respective proposition is presented.* In other words, an agent's knowledge and beliefs primarily concern *concepts*, i.e. *constructions* of propositions.

So far we have considered *implicit* (i.e. *intensional*) attitudes; relations to propositions. Such attitudes are actually rather non-realistic attitudes of an idealistic agent with unlimited inferential capabilities, i.e., of a logical/mathematical genius. In Artificial Intelligence the distinction between *explicit* and *implicit* knowledge plays an important role. Explicit knowledge is that what is explicitly recorded in a knowledge base, whereas implicit knowledge is that what can be obtained from the former by means of a particular (proper, correct) inference machine. It is a well-known fact (due to Gödel's incompleteness results) that regardless of how sophisticated an inference machine is, there is never going to be an absolutely perfect machine that would be capable of mechanically deducing *all* the logical consequences, all the implicit knowledge. These facts trigger the question whether a more fine-grained, non-intensional analysis of attitudes might be at hand. The problem was brought up al-

ready by Frege, and since then logicians strive after structured meanings that would adequately render the identity of beliefs in attitude contexts. Carnap's attempts at solving the problem by means of the notion of intensional isomorphism have been proved to be inadequate by Church.[9] Cresswell answers in the affirmative way, dubbing the problematic contexts *hyper-intensional*, and proposing what he calls *structured meanings* as solution. In TIL we agree both with hyper-intensional individuation of attitude relata and structured meanings. However, our manner of fleshing out the latter is different. We cannot accept Cresswell's, and others', assumption that tuples are structures in the desired sense, and instead offer compositions and closures as structured meanings.[10]

Besides the above *implicit* attitudes to propositions we have to consider *explicit attitudes* to the constructions of propositions, i.e., to hyper-propositions. Implicit attitudes are closed under the relation of logical consequence and the transfer between *de dicto*/*de re* attitudes is realisable under some additional conditions. However, the natural demand that the believer, if confronted with a sentence describing the attitude attributed to him, would, and rationally must, assent to it, does not have to be met. The agent in question does not have to be able to perform the respective inferences, his logical/mathematical skills being limited. In other words, he need not (explicitly) know that he (implicitly) knows/believes that …; the agent may behave, so to say, logically irrationally.

In order that the agent shall behave rationally, the proposition that has been obtained by the agent's applying a point of view has to be presented by the reporter in a way *synonymous* to the way the agent would use. Repeating briefly:

> *Synonymous expressions* are such expressions which express exactly the same construction of the denoted object.[11]
> *Equivalent expressions* are such expressions which denote one and the same object *o* possibly *via* distinct constructions that are equivalent in the sense of all constructing *o*.

Explicit attitudes are defined as relations-in-intension to a *construction* of a proposition. They are of type $(o\ \iota\ *_k)_{\tau\omega}$, k being mostly equal to 1. The

[9] See Carnap (1947), Church (1954), Cresswell (1985).

[10] For tuples as structures see Jespersen (2002).

[11] Actually, synonymous expressions express one and the same *concept*, see Chapter 5.

(schema of) analysis of *de dicto* and the respective *de re* attitude be as follows (here we have: $B^e/(o\,\iota\,{*_1})_{\tau\omega}$, Ch/ι, $F/\iota_{\tau\omega}$, $G/(o\iota)_{\tau\omega}$, x ranging over ι).

De dicto case:

(8) $\lambda w\lambda t\,[{}^0B^e_{wt}\,{}^0Ch\,{}^0[\lambda w^*\lambda t^*\,[{}^0G_{w^*t^*}\,{}^0F_{w^*t^*}]]\,]$

(${}^0F$ *de dicto* – '… that the *F* is *G*')

The *de re* case is rather more complicated. We first analyse the passive variant, and then by using valid β-reduction we obtain the analysis of the active variant. To this end we have to construct the property of being *explicitly* believed by Charles to be a *G*. The first attempt of doing so in an analogous way as in the implicit case is *not* correct:

$[\lambda w\lambda t\,\lambda x\,[{}^0B^e_{wt}\,{}^0Ch\,{}^0[\lambda w^*\lambda t^*\,[{}^0G_{w^*t^*}\,x]]]]$.

For, there is a technical problem here. The variable x occurs as a constituent of a construction that has been trivialised: ${}^0[\lambda w^*\lambda t^*\,[{}^0G_{w^*t^*}\,x]]$. Hence the variable x is o-bound here and cannot become λ-bound, cannot be so simply abstracted on. The philosophical explanation: Those who are explicitly believed by the agent Charles to be so-and-so are completely wrapped within his perspective, and thus as if not accessible. To overcome the problem, we use the function *Sub*(stitution)[12] of type $({*_n}{*_n}{*_n}{*_n})$, which, when applied to a triple of constructions C_1, C_2, C_3, returns the construction C that is the result of correctly substituting C_1 for C_2 in C_3. The property of being explicitly believed by Charles to be a *G* is a criterion that can be applied to individuals. Hence a trivialisation of the individual *v*-constructed by x has to be substituted for x: $[{}^0Tr\,x]$ serves this purpose, $(Tr/({*_1}\,\iota)$—the function that returns the trivialisation of its argument). The result of

$[{}^0Sub\,[{}^0Tr\,x]\,{}^0x\,{}^0[\lambda w^*\lambda t^*\,[{}^0G_{w^*t^*}\,x]]]$

is the *construction* $C/{*_1}$ to which Ch(arles)$/\iota$ is related by the explicit belief $B^e/(o\iota{*_1})_{\tau\omega}$. The construction C *v*-constructs the proposition that the x is a *G*.

This technical 'trick' makes the variable x free to be λ-abstracted on, and the property of being explicitly believed by Charles to be a *G* gets the analysis:

$\lambda w\lambda t\,\lambda x\,[{}^0B^e_{wt}\,{}^0Ch\,[{}^0Sub\,[{}^0Tr\,x]\,{}^0x\,{}^0[\lambda w^*\lambda t^*\,[{}^0G_{w^*t^*}\,x]]]]$.

[12] For details see Tichý (1988), Materna (1997), Duží (2006).

The sentence 'The *F* is explicitly believed by Charles to be a *G*' is analysed as

(9) $\lambda w\lambda t\,[[\lambda w\lambda t\,\lambda x\,[{}^0Be_{wt}\,{}^0Ch\,[{}^0Sub\,[{}^0Tr\,x]\,{}^0x\,{}^0[\lambda w^*\lambda t^*\,[{}^0G_{w^*t^*}\,x]]]]]_{wt}\,{}^0F_{wt}]$

or β-equivalently

(9β) $\lambda w\lambda t\,[\lambda x\,[{}^0Be_{wt}\,{}^0Ch\,[{}^0Sub\,[{}^0Tr\,x]\,{}^0x\,{}^0[\lambda w^*\lambda t^*\,[{}^0G_{w^*t^*}\,x]]]]\,{}^0F_{wt}]$.

Note that since the variable *x* occurs now in the extensional *de re* context the β-reduction substituting ${}^0F_{wt}$ for *x* is valid:

(9ββ) $\lambda w\lambda t\,[{}^0Be_{wt}\,{}^0Ch\,[{}^0Sub\,[{}^0Tr\,{}^0F_{wt}]\,{}^0x\,{}^0[\lambda w^*\lambda t^*\,[{}^0G_{w^*t^*}\,x]]]]$
${}^0F$ *de re* – '… believes *explicitly of* the *F* that it is a *G*.

The resulting construction (9ββ) is expressed by the active variant of the *de re* attitude:

Charles believes of the F that he/she/it is a G.

The anaphoric reference *he/she/it* expresses the free variable *x* for which Trivialisation of the individual playing the role of the *F* is substituted.[13]

Note that (9β, 9ββ – *de re*) are again not equivalent to (8 – *de dicto*), namely for the same reasons as set out above (existential presupposition, and substitutability of a co-occupied office in the *de re* case, unlike the *de dicto* case), but this time even the additional 'fresh premise' of the agent's knowledge of the occupancy of the office *F* does not make it possible to perform logical transformations between *de dicto* and *de re*. These more realistic explicit attitudes are, the other way around, also non-realistic: This time our agent is deprived of *any inferential capabilities.*

We might introduce some classes of axioms and rules to characterise explicit attitudes, which would serve to tune the agents into various categories, according to particular inference rules the agents are capable of using, beginning with the simplest ones (e.g. the commutativity of conjunction and disjunction) up to some sophisticated rules. But this epistemic task, characteristic of resource-bounded belief revision, is out of the scope of the present study.

We have seen that using particular points of view is a way of disambiguating our statements. If we claim, 'Charles believes that nuclear power is dangerous', we have to take into account the fact that Charles'

[13] For details on the analysis of sentences with anaphoric reference see Duží (2006a).

belief cannot be absolute in the sense of nuclear power's being absolutely dangerous. Charles probably believes so, because he has applied the criterion of physical and chemical properties of nuclear power, or any similar viewpoint. So he actually believes that nuclear power is dangerous from the point of view of its explosive power (or any other criterion of his evaluation). The question is whether an agent can ever ascribe to the viewed object any such vague property like being dangerous absolutely. A possible conclusion might be: The agent always applies some point of view to evaluate an object using a particular criterion—thus obtaining a proposition—when arriving at his attitude to the object of his belief. The question is, however, whether the resulting proposition is referred to in a faithful, authentic way. We have shown that there are two kinds of an attitude report, namely *de dicto* and *de re*, and two ways of understanding (analyzing) them: as implicit (propositional) vs. explicit (hyperpropositional). Which of them, if any, is logically entailed by applying a particular viewpoint, and which can be taken as just being justified or explained by the viewpoint application?

At this point, a dialogue between a reporter *R* and an agent *A* might again help to elucidate our ideas:

R: Do you believe that the Pope is an old man?
A: No, I don't.
R: Why not? Surely you know that the Pope is already over 80.
A: Yes, from the point of view of age, the Pope is an old man, but from the point of view of his vitality, he is not an old man.

Now, a conclusive report on agent's attitudes to which the agent does assent can be *de dicto* only:

> The agent *a* believes that the Pope is an old man from the view-point of his age, but that he is not an old man from the view point of his vitality.

Analysis *de dicto*:

$$\lambda w\lambda t\,[[{}^0B^e_{wt}\,a\,{}^0[\lambda w\lambda t\,[[{}^0V_{wt}\,{}^0Age\,{}^0Pope_{wt}] = \lambda w\lambda t\,[{}^0Old_{wt}\,{}^0Pope_{wt}]]]] \wedge$$
$$[{}^0B^e_{wt}\,a\,{}^0[\lambda w\lambda t\,[[{}^0V_{wt}\,{}^0Vital\,{}^0Pope_{wt}] = \lambda w\lambda t\,\neg[{}^0Old_{wt}\,{}^0Pope_{wt}]]]].$$

Here the equality $=/(\mathrm{o}\ \mathrm{o}_{\tau\omega}\ \mathrm{o}_{\tau\omega})$ is a relation between propositions. The reporter does not accuse the agent of any paradoxical belief. Due to the vagueness of an empirical property like *Old* the borderline between

a positive and a negative ascription is not sharp and can depend on a given viewpoint.

Actually, no other report of the situation would be fully adequate in this case. If there is another premise to the effect that the agent knows that Ratzinger is the Pope, the reporter might use the explicit *de re* way. If the reporter uses any other *equivalent* way, the agent does not have to assent to his report, because he only 'implicitly' knows, believes, etc., so-and-so, without being explicitly aware of it.[14]

There are still some other problems connected with disambiguation. Simple expressions do not have to represent simple constructions, i.e. just trivialisations of denoted properties. Moreover, empirical properties are usually to some extent vague. What does it exactly mean that somebody is old or that something is dangerous? When would we say that small tree is a shrub? These problems may be partly overcome by using an *ontological definition*,[15] i.e. a *complex* construction of the property in question. For instance, the property of being a cat can be defined more precisely using a biological definition: A cat is a domestic carnivorous feline, etc. But in that case there is another question as to whether this complex construction constructs exactly the property which the agent has in mind when using a simple concept. Logic, which cannot take into account psychological and pragmatic perspectives, cannot answer this question. We will deal with ontological definitions in the following Section 5.

5. Concepts and ontological definitions. The conceptual viewpoint

A special case of using various points of view concerning one and the same object can be described only after explaining how our theory explicates the term *concept*.

In Materna (1998) it has been shown that set-theoretical explications of this term are connected with insurmountable difficulties. Concepts should be construed as *structured, complex* objects, something like *abstract procedures* that are encoded by expressions and identify those objects (if

[14] Therefore, a precise analysis of the *reasons* for particular attitudes, i.e. the analysis of applying points of view and evaluating their criteria, and properly reporting such a situation, might to some extent explicate the role of fuzzy logics.

[15] See Materna (1998).

any) that are denoted by these expressions; cf. the Frege-Church schema in §2. Obviously one of the best options is *construction* as defined above. Thus *concept* can be defined (roughly) as a *closed construction*, i.e., a construction that does not contain free variables.

There are some sophisticated steps that make it possible to ignore some distinctions between constructions w.r.t. the notion of *concept*. As an example consider the constructions

$\lambda x\ [^0{>}\ x\ {}^0 0]$
$\lambda y\ [^0{>}\ y\ {}^0 0]$

and observe that both construct the class of positive (real) numbers. Being distinct, they all the same represent one and the same concept, they are not distinguishable from the conceptual point of view, because the procedure for arriving at the set is the same regardless of which variable is used here.[16]

Let X be any object except a construction. The construction ^0X will be called a *simple concept* of X. Finally, *conceptual systems*[17] are based on a set of simple concepts (*primitive concepts* of the given system) and the set of *complex concepts* that can be derived from these primitives using variables, trivialisation, composition, closure and double-execution. The types over which the variables can range in the given conceptual system are called *pre-concepts* thereof.

Now let C and C' be two *equivalent* concepts, at least one of which is not simple. Being equivalent to C' means that C identifies (constructs) the same object o as C'. This time, however, we cannot speak about a partial characterisation of o (the constructed object o is fully determined by C as well as by C'). All the same we probably feel that we are justified in saying something like, *From the viewpoint of C the object 'behaves' so and so, whereas its 'behaviour' from the viewpoint of C' is ... .*

C and C' can be non-empirical or empirical concepts. The former case is simpler. Compare the following concepts (x, y range over real numbers):

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[16] This does not mean that all pairs of equivalent concepts (those which construct one and the same object) would represent just one concept. Only the so-called *quasi-identical* constructions do so. Roughly, they are constructions that differ only by being unnecessarily η-expanded, or constructions that are α-equivalent, i.e., that differ only by 'renaming' λ-bound variables. See Materna (2004).

[17] Needless to stress that conceptual systems defined in this way differ from what Putnam has called *conceptual scheme.*

$\lambda x\ [^0{>}\ x\ ^0 0]$,
$\lambda x\ [^0\exists\ \lambda y\ [^0{=}\ y\ [^0\sqrt{}x]]]$.

Both concepts construct the set of positive real numbers. The first of them does it 'from the viewpoint of', say, ordering, the second takes into account the possibility of extracting the square root. Such cases are simple in that one can always *prove* the equivalence of the respective concepts.

More problems arise when both concepts are empirical. Then, in general, their equivalence cannot be demonstrated simply by offering a *mathematical/logical proof*. True, sometimes it is possible: when both concepts are members of the same conceptual system (i.e. determined by the same set of simple concepts) then one of them may arise through equivalent transformation, which is a matter of logic. The problematic (and more interesting) cases are those ones where the concepts belong to *distinct* conceptual systems. We can imagine the situation where one concept is a member of a conceptual system of a fragment of physics and the other one belongs to a conceptual system of some fragment of chemistry; do not forget that not only primitive concepts of both systems but also their pre-concepts, i.e., types, can differ. This would be the case, for example, if we would ask: What is *water* from the viewpoint of physics, and what is *water* from the viewpoint of chemistry?

But might such a question not be futile? Isn't the 'physically viewed' water the same object (only viewed from another 'perspective') as the 'chemically viewed' water? And what about the concept of water that underlies the expression 'water' of *ordinary* language?

To demonstrate equivalence in our example (which can be easily generalised) we would have to eliminate such cases where we would have to state either that something is water according to the 'physical definition' but not according to the 'chemical definition' or *vice versa*. If these cases were not eliminable then it would mean that there were some property, viz. *being water* some features of which are determined by one of the concepts and, maybe, some other features by the other concept; partiality of the characterisation would return, and the property *being water* would be what underlies natural language, whereas 'physical water' and 'chemical water' would be some other properties.

A special case of equivalence of empirical concepts can be stated when this equivalence is *analytical* and can be transformed to a *logical equivalence*. In terms of conceptual systems this case can be described as a transition from a conceptual system **CS** to another **CS'**, the latter being

a '*decomposition*' of the former.[18] The following example shows, *inter alia,* what is meant by decomposition.

Let the initial conceptual system contain the primitive concept ^{0}the_father_of (*father/*$(\iota\iota)_{\tau\omega}$). Let the 'decomposing' conceptual system contain concepts ^{0}a_parent (*parent/*$(o\iota\iota)_{\tau\omega}$) and ^{0}male (*male/*$(o\iota)_{\tau\omega}$) (and besides some logical concepts, among others the 'singulariser' $^0\iota$, $\iota/ (\iota(o\iota))$). Compare the concepts

^{0}the_father_of

and

$\lambda w \lambda t\, \lambda y\, [^0\iota\, [\lambda x\, [^0\wedge\, [^0\text{a\_parent}_{wt}\, xy]\, [^0\text{male}_{wt}\, x]]]]$.

Both concepts identify the same intension: to see this it is sufficient to know English. The second concept makes explicit the requisites of this intension. Therefore it is the second concept (not the first!) that makes it possible to *logically prove* that the sentence 'Every father is a man' is a logical tautology.[19]

What connects this example with the problem of *distinct points of view of one and the same object*? The object in question is here the individual role *the father of X*. Can we say that by using the system where the respective concept is primitive we apply another viewpoint on this role than by using the above decomposition of the former system? In a sense we can: in the former case no special viewpoint, no special 'perspective' is assumed (this is always the case when simple concepts are used); in the latter case the viewpoint is given by the attributes *being a relative* (whose one value is *being a parent*) and *sex/gender* (whose one value is *male*).

Also in such cases, i.e., in the cases of viewpoints given by conceptual distinction, the scheme of the type(s) of *point of view* can be preserved, this time in the extensional version:

$V/(o_{\tau\omega}\ \alpha\ \beta)$,

where α, the type of the criterion, is the set of attributes the constructions of which are members of the content of the given concept. In our example we could perhaps write schematically

[18] For details see Materna (2004).

[19] So decomposition is to make explicit some constructions needed for particular inferences.

[^0V { } the-father-of] (the first case)

and

[^0V {relative, sex} the-father-of] (the second case).

The resulting proposition in this second case is *The father of X is a male parent of X*. The first case does not lead to any proposition, as the set of attributes (the criterion of evaluation might consist of) is empty.

We have seen that the trivialisation of an object *X*, i.e. ^0X, delivers immediately *X* without any perspective. Hence we cannot logically deduce any non-trivial consequences from analyses using trivialisations of the denoted objects. For instance, from [^{0}*Prime* ^{0}7], where *Prime*/(οτ) is the set of prime numbers, no interesting consequences can be inferred. But if we know what 'prime' means, we also know that the following arguments are valid:

7 is a prime number.
Hence 7 is divisible by 1 and 7 only.
Hence 7 is not divisible by 2, or by 3, or by 5, etc.

This means that a simple expression E is usually not connected with the trivialisation of the object *O* that E denotes, but that a *complex concept* that identifies the object *O* is assigned to E as its meaning. Therefore we define:

Each complex nonempty concept *C* is an *ontological definition* of the object *O* constructed by *C* (read: *concept C defines the object O*).

Example:
An ontological definition of (the class of) prime numbers/(οτ) is:

$\lambda x\ [[^0\text{Nat}\ x] \wedge [^0\text{Card}\ \lambda y\ [[^0\text{Nat}\ y] \wedge [^0\text{Div}\ x\ y]]] = {}^0 2]$

(Here: Nat/(οτ) is the class of natural numbers, Card/(τ (οτ)) is the function *number of elements*, Div/(οττ) is the numerical relation *being divisible by*.)

This is not the common way of using the term 'definition'; there is no *definiendum* and *definiens*. By a 'definition' we usually understand the following schema:

Expression E_1 (*definiendum*) $=_{df}$ expression E_2 (*definiens*).

From the logical point of view, this is a *linguistic definition*. It associates expression E_1 (which is usually, but not necessarily, a simple expres-

sion that has not been used in the language so far) with the meaning of the expression E_2.

Examples:
Primes $=_{df}$ Numbers that have exactly two factors
Cat $=_{df}$ Domestic carnivorous animal, a feline, …
The father of $=_{df}$ the male parent of
Bachelor $=_{df}$ an unmarried man

Note that ontological definitions do not obey the schema of *definiendum* = *definiens*; they are not definitions of terms (i.e. meaning assignments), but of objects. Hence ontological definitions are prior to linguistic ones. An ontological definition does not assign a meaning to an expression. Instead it defines a particular kind of object and does so by circumscribing a concept that identifies the object. At the same time, concepts are the senses of expressions. In other words, linguistic definition assigns a (new) meaning to E_1, namely the ontological definition of the object denoted by E_2, i.e., it makes the two expressions *synonymous* on the basis of linguistic convention.

Using a conceptual system C in which ^{0}prime is a primitive concept and which does not contain any of the simple concepts ^{0}Div or ^{0}Card as primitive, we could not use the above ontological definition of primes. But in this case we certainly do understand 'prime' just in the sense of this definition, which means that we use some more decomposed conceptual system C' (containing, at least, ^{0}Nat, ^{0}Div and ^{0}Card). Of course, a deeper decomposition is still possible, which would, for instance, enable us to define the set of natural numbers on the basis of, say, the Peano axioms. Thus simple expressions often do not express primitive concepts (i.e., trivialisations of the denoted object) of a given conceptual system, but *complex concepts* composed of simpler primitive concepts of a finer decomposed conceptual system, see also Materna (2004).

An *a priori* discipline, logic, cannot substitute the role of empirical disciplines, like linguistics, biology, physics, etc. Hence logic cannot provide, e.g., an ontological definition of the property of being a cat, nor can it decide whether the respective definition is 'correct' in the sense of identifying exactly the same property as the one which is understood by 'cat' in a natural language such as English. Nor can logic impose pragmatic features and decide which conceptual system an agent has used.

Instead, one of the tasks of logic is to build up particular logical (axiomatic) theories. If a set of ontological definitions is furnished by

a linguist (or biologist, physicist, etc), logic provides rules of inference enabling us to deduce logical consequences of these definitions (axioms). Hence having particular ontological (and linguistic) definitions at our disposal, we can look at primes, cats, water, etc. from particular (mathematical, biological, physical, etc) points of view; this time we can obtain either a partial characterisation or even a total characterisation of a given intension/extension.

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Errata

In "Points of View from a Logical Perspective (I)" (*Organon F* 13, 2006, No. 3, 277 – 305) we introduced the notion of a requisite, an analytical relation between intensions. We defined the case of a property being a requisite of another property (*Req*^{pr}), or of an office (*Req*^{of}). And we also defined typical properties for a property or for an office. Unfortunately, we did not take into account the possibly properly partial character of the respective intensions. Since the requisite relation is valid independently of the fact whether a property is instantiated or an office occupied, we have to use the property of propositions (of being true), $True/(o\ o_{\tau\omega})_{\tau\omega}$, where $[^0True_{wt}\ P]$ *v*-constructs the truth-value **T** iff P_{wt}, otherwise **F**. The definition has thus to be corrected as follows:

Let *p*, *q* be variables of 1st order, ranging over properties $(o\alpha)_{\tau\omega}$, *c* a variable ranging over offices $\alpha_{\tau\omega}$, *x* a variable ranging over α. We define:

$Req^{pr} =_{df}\ \lambda pq\ \forall w \forall t\ \forall x\ [[^0True_{wt}\ \lambda w \lambda t\ [q_{wt}\ x]] \supset [^0True_{wt}\ \lambda w \lambda t\ [p_{wt}\ x]]]$
(*p* is a requisite of *q*)

$Req^{of} =_{df}\ \lambda pc\ \forall w \forall t\ [[^0E_{wt}\ c] \supset [^0True_{wt}\ \lambda w \lambda t\ [p_{wt}\ c_{wt}]]]$
(*p* is a requisite of the office *c*)

$TP^{pr} =_{df}\ \lambda pqe\ \forall w \forall t\ \forall x\ [\neg[^0True_{wt}\ \lambda w \lambda t\ [e_{wt}\ x]] \supset [[^0True_{wt}\ \lambda w \lambda t\ [q_{wt}\ x]] \supset [^0True_{wt}\ \lambda w \lambda t\ [p_{wt}\ x]]]]$
(*p* is typical of *q*, unless *e*)

$TP^{of} =_{df}\ \lambda pce\ \forall w \forall t\ [[^0E_{wt}\ c] \supset [\neg[^0True_{wt}\ \lambda w \lambda t\ [e_{wt}\ c_{wt}]] \supset [^0True_{wt}\ \lambda w \lambda t\ [p_{wt}\ c_{wt}]]]]$
(*p* is typical of the office *c*, unless *e*)

where $E/(o\alpha_{\tau\omega})_{\tau\omega}$ is the *property of existence*, and *e* ranges over 'exceptions', 'the *unless properties*'$/(o\alpha)_{\tau\omega}$, the so-called *guards* of the rule in Artificial Intelligence.