STATE

REFORMULATING TICHÝ'S CONCEPTION OF BARE INDIVIDUALS

Jiří Raclavský

ABSTRACT: A bare individual was conceived by Tichý as an individual such as if the property the individual instantiates is non-trivial (contingent), it is possible for the individual to lack it (and still be the same individual); and for any trivial property (i.e. property with constant extension) that it cannot lack this kind of property. The exact readings of Tichý's original formulations of bare individuals are subjected to a detailed analysis to reveal that any of them is refutable by means of Cmorejian objection that there exist contingent properties which are partly essential (i.e. there exists an individual which cannot lack such property). To avoid such (valid) objection, the present paper attempts to rebuild Tichý's definitions into viable ones.

KEYWORDS: Bare particulars, individuals, antiessentialism, properties, intensional logic.

According to a certain view on individuals (or particulars), an individual that has lost certain substantial (essential) property ceases to be the very same thing (and happens to be a distinct entity). This view is shared by many philosophers and their doctrine is called essentialism. Using theoretical framework of possible worlds, one has to ask whether this doctrine is acceptable because when certain thing is an „F" and then it loses that „F" we still attribute something to the very same object. Thus a *bearer* of properties should be conceived *as conceptually distinguished from*, i.e. not to be confused with, *properties* it instantiates or lacks (or: can instantiate, can lack), thus construed in a certain sense resistant to the changes of its properties.

The very idea of possible worlds presupposes that various properties are distributed through a collection of individuals. Conceivable world alternatives should then share the same set of individuals, i.e. bearers of properties, which does not change from one alternative to another. Clearly, individuals only change properties they instantiate in this or that possible world. Subsequently, it seems that to dismiss essentialism in favour of the doctrine of *bare individuals* is entirely necessary in order

ORGANON F **15** (2008), No. 2, 143 – 167

to save the original idea for which the possible-worlds framework was set up. In Pavel Tichý's variant of this conception, no non-trivial property (i.e. property like „being a human") can be instantiated by an individual by virtue of necessity. On the other hand, an individual (or any other entity) cannot lack any trivial property (such as the property „being identical with itself").

In this paper, I am not going to attack any of the standard forms of essentialism; I simply share the antiessentialistic views and arguments presented already by Pavel Tichý. Rather, I want to defend his doctrine of bare individuals against criticism based on Pavel Cmorej's reference to partly essential properties which are non-trivial (contingent) but there exists an individual that cannot lack them. So this disproves Tichý's formulation of the conception of bare individuals. Nevertheless, Tichý's original definition of bare individuals can be modified into its more proper version. Key formulations (as well as classifications of properties) will be provided in exact terms to avoid potential misunderstandings.

Tichý's antiessentialism

Intensional framework (of scientific inquiry) may be conceived as a proposal going beyond extensionalistic framework which was built upon the basis of quantified predicate logic. It does so not by throwing away fundamental ontological principles underlying predicate logic. For instance, the well-known axiom which can be expressed as 'every entity is identical with itself' is accepted without any exception. Since no entity without self-identity is rationally conceivable, the extension of the property „being self-identical" is independent of thinkable circumstances. Clearly, all entities happen to instantiate this property in all possible worlds. This modal invariability of property, the absence of contingent dependency on states of particular possible worlds, is a remarkable feature of properties Tichý called trivial (Tichý borrowed the term from Alvin Plantinga). We define the family of such properties and the family of properties complementary to them as follows (for formal definitions see the appendix):

being a *trivial* property $f =_{df}$ being a property such that either in every possible world w' it is true that the extension of f in w' is identical with the extension of f, or in no possible world w'' there exists a class s identical with the extension of f in w''
being a *non-trivial* property $f =_{df}$ not being a trivial property f

To excogitate the extension of any trivial property in whichever arbitrary possible world one needs not to go out to investigate the contingent state of that world. It is sufficient enough to put one and two together and state, by mere deduction from basic principles, exactly which class is the extension of a given property in that particular world. It is apparent that non-trivial properties do not embody such a remarkable feature. But their importance lies in the very reason why the possible-worlds framework was adopted. To know the extension of a non-trivial property one has to examine individuals for instantiating of this property in the particular world.[1] Thus we impose restrains upon the distinction between *trivial* and *non-trivial properties*, which can be also conceived as *non-empirical* and *empirical properties* (or *constant* and *contingent*).

What is an *individual*? We will limit ourselves to the view that – in accordance with the original Latin meaning of the term 'individuus' – it is a simple (i.e. non-divisible) entity, thus it is an entity at the very bottom of our (complex) ontology. It is an individual entity in the sense that it is unique, i.e. numerically distinct from any other entity.[2] The term 'individual' is often interchanged with the term 'particular'. Recalling here the original Latin meaning again, a *particular* is an individual thing, a particular instance, of certain kind.[3] For instance, Alan, who is actually a man, is an individual instance of the kind (universal, property) „man".

Being a unique instance of a certain kind may be construed materialistically in the sense that it is a real definite thing under consideration, i.e. that we do not leave the realm of concrete beings. Nonetheless, there is also another construal of particulars, namely that a particular is a unique 'thing-to-be', a special kind of entity which is quite different from whatever material things there are. In our 'technical' construal this second notion of a particular will be understood as *individual office*. Since

---

1 This is not to insist that checking up of individuals for instantiating of every non-trivial property is necessary. As soon as we know that Alan is 180 cm high and Beth is 170 cm high (in a certain world), we do not need to perform an empirical test in order to state that Alan possesses the property „being taller than Beth". But this fact does not change the empirical character of the property „being taller than Beth" as it is evident in the case the facts of Alan's and Beth's actual heights are not known. To conclude the point, it is theoretically possible to diminish the number of empirical tests needed for full description of the state of a given world.

2 Within our intensional framework we do not conflate, for example, numbers with individuals. But any entity, including numbers, is individual, unique.

3 No doubt, various philosophers have different notions of particulars.

individuals and individual offices will be strictly distinguished, we will avoid using the ambiguous term 'particular'. The bearers of properties shall be called just 'individuals'.

To know whether the sentence expressing the state of affairs consisting in Alan's being a man, namely 'Alan is a man', is true (and thus expressive of a real fact, the state of affairs that obtains), one has to go out into the real world and conduct a test for „being a man" on Alan. No doubt, Alan is not a man by virtue of necessity. Therefore, an extensional claim to the effect that the truth-condition of the respective sentence should be explained in the sense of mere class membership is inadequate – for class membership is entirely a non-contingent matter. Intensional framework incorporates rather the idea that there are conceivable circumstances, *possible worlds*, in which, for example, Alan is not a man, but a woman instead. The truth-conditions of the sentence 'Alan is a man' are identified with the class of possible worlds in which Alan is a man. By means of extrapolation, *property* is not identified with any particular class but with a *function from possible world to classes*.

It needs to be pointed out that whether Alan is (or is not) a man is quite independent of Alan's real nature *qua* individual. In those possible worlds in which he is not a man, he is still the very same individual that we call 'Alan'. Contrary to this view, some representatives of common sense insist that Alan's ceasing to be a man and becoming an instance of womankind substantially affects Alan's own identity. In accordance with the wide sense of identity underlying such reasoning, somebody would surely subscribe to this claim – when Alan undergoes, perhaps under some physical pressure, a sex-change operation, he may be rather afflicted in regard to his personal self-appreciation, feeling now as a different individual. In logic and intensional metaphysics we had better avoid such a notion of identity and restrict identity to mere self-identity in the sense declared above. Thus no internal psychical state of Alan is taken into consideration (moreover, Alan conceived as a mere individual lacks soul as his constitutive part). On the other hand, some other representatives of common sense would agree with us that Alan, after a sex reassignment surgery from male to female (perhaps as a result of his own decision), is still the very same individual as before (even if he calls himself 'Beth' now) and that the only thing which is different is his differrent looks, i.e. the possession of different properties than before. From this thus follows that antiessentialism is involved within the sphere of common sense views too. Hence,

there should be no risk of material inadequacy of our theoretical explication when we prefer just *antiessentialism*. Let us remind ourselves that the antiessentialistic view just sketched involves that being an instance of this or that kind does not change the individual which happens to change certain properties. Being this or being that (to be an instance of this or that kind) is thus perfectly conformable with antiessentialism. One may also realize that Alan can indeed easily explain to his mother – perhaps insisting on the essentialistic thesis that becoming an instance of another kind affects the very nature of an object undergoing such change – that he is really identical with her descendant she hitherto called 'Alan'.

Things are slightly less easy when we want to defend antiessentialism against those who conflate individuals with individual offices. Common people are practical beings using language to speak about real-material things. When we, as theoreticians, ask them philosophical questions such as what is a number?, what is a class?, etc., they are urged to provide answers by means of which such entities are identified with visible, palpable, 'real' things, e.g. numerals or set diagrams. The 20th century essentialism seems to be conformable with those non-theoretical respondents. So when I talk about my watch (for definiteness, „the watch of J.R."), they construe the object I am talking about as inevitably a real, palpable, visible, material thing, a specific individual. In everyday life such construal is frequent and reasonable: there are practical reasons why people do not make fools of each other in speech, why they presuppose that there exists a definite material object, a piece of metal, which serves me as a device for time measurement. But being watchless and speaking about my watch, my sentences are still about a definite and perfectly graspable subject matter: „my watches". For another example, I can talk about my newest Rolls-Royce despite the fact that I am actually not an owner of any car of that sort. By way of illustration, let us introduce the famous and more striking examples – the U.S. president and the only king of France. It would be absurd to identify the U.S. presidentship with any particular individual who happens to fill this post in the actual world (and the present moment). The chronology of facts in the actual world shows that the claim of extensionalists, reducing the U.S. presidency to mere individuals, such as G. W. Bush (Bill Clinton etc.), is quite odd. When we say that nobody is the king of France, we do not talk nonsense for allegedly non-existent individual but we express a perfectly sensible statement. This statement has a clear subject matter:

the thing which can be filled by this or that individual. Those individual offices, things occupiable by individuals, such as „the U.S. presidentship" or „my watch" are genuine entities we can talk about. Intensional framework aptly individuates *individual offices as* (partial) *functions from possible worlds to individuals* (note that a mapping of that sort assigns to any particular world an individual, not a singleton).

Let us return to the problem of essentialism. The individual, let us call it 'H', which actually happens to be my watch instantiates several properties such as „being made in Switzerland", „being chrome", „being a time measuring device", etc. Of course, any suitable individual which is a time measuring device can happen to become my watch, to fulfill that office. Having the property „being a time measuring device" is a necessary, *required condition* for that. On the other hand, having the property „being made in Switzerland" is not strictly required, it is optional. My watch as a thing-to-be, an individual office, consists just in such required conditions as that one mentioned above. Thus it cannot lack them at all, they are *internal* to it. On the other hand, for H being the very same individual no such condition is required. Thus none of them is its constitutive part. When I take H – an individual to which I may refer concurrently by means of the description 'my watch' – to a locksmith and ask him to remake this piece of metal into a key and he will do it, the individual H would be still identical with H. Only H would become an instantiation of another property (which is not constitutive of it either), namely „not being a time measuring device" (it is now surely lacking the property „being a time measuring device"). Therefore, H is still the same individual. „Being a time measuring device" or „not being a time measuring device" are features which are *external* to this very individual.[4]

Now we can understand why essentialists insist that my watch cannot lack the property „being a time measuring device" for there is just one sense of their claim on which we may agree with them. By the very definition of „my watch", this individual office is constituted, *inter alia*, by the condition „being a time measuring device". Consequently, 'My watch (here referring to the office) is a time measuring device' is

[4] Of course, if H was already made in Switzerland, the property „being made in Switzerland" belongs inevitably to its *history* and is not dispensable from it now. (However, it is still thinkable that H is in fact not made in Switzerland; to check whether it is, or it is not, made in Switzerland, one must inquire about the actual state of the external world).

a necessarily true sentence. This must not be confused with an entirely distinct use of the description 'my watch', in particular, a reference to a certain piece of metal, e.g. the individual H. In such case, the claim 'My watch is necessarily a time measurement device' is not tautologically true, but only contingently true. Therefore, one must carefully *distinguish between a reference to an individual and a reference to an individual office*.

For another example, whenever we claim that the president of the U.S. is necessarily a president, that it cannot become a non-president, we sensibly talk about conditions in the definition of „American presidency", not about G.W. Bush who can be neither president, nor American. On the other hand, to insist that H must be a time measuring device is thus to deny the obvious: a strong stroke of a hammer can easily deprive H of the property „being a time measurement device". This means that I cannot then properly refer to H by means of the phrase 'my watch'. It is apparent that once I ask the locksmith to give that individual back to me, I cannot argue that he should give me my watch because the individual H (I wish to retrieve) is now not a watch anymore. But this piece of metal, H, still happens to be originally mine – essentialism notwithstanding. Significantly enough, once individuals and individual offices are clearly distinguished, the strange consequences of essentialism automatically evaporate.

As has been already mentioned, such a strong argumentation against individual essentialism (as well as the defence of individual offices) was already stated by Tichý himself (unintentionally, but naturally, I have probably used some of his claims in this section above). I recommend to the reader his article 'Individual and their Roles' (Tichý 1994, reprinted in Tichý 2004, especially sections 1 – 3), originally an introduction to his unpublished monograph 'Introduction to Intensional Logic', finished in 1976. Then you may read Tichý's polemic with Plantinga's essentialism (Tichý 1972). For Tichý's refutation of genealogical essentialism ('necessity of origin') or the wooden/ice table puzzle consult section IV (Tichý 1983) (see also Tichý 1988, §36).

One may wish to find a brief and pregnant formulation of antiessentialistic credo which would incorporate considerations given above. Such an outline stands in the focus of the following investigation. As soon as we deal with Tichý's writings, we may find exactly *four formulations of the conception of bare individuals*, all of them similar to one another. The first one is contained in the oldest article discussing the topic 'On

Describing' written in 1973, i.e. in the paper that was unknown until its quite recent publishing.[5] Among other, we read:

> Venus is, of course, bare not in the sense of lacking properties: indeed, for any relevant property X, Venus instantiates either X or non-X. ... Venus is bare because for any non-trivial property X it happens to instantiate, Venus might conceivably have lacked X without thereby ceasing to be the same thing. They are of course, sundry trivial properties ... which are necessarily instantiated by Venus (Tichý 2007, 424).

The second oldest paper discussing bare individuals 'Individuals and their Roles', was published also with delay:

> It [individual; J.R.] is bare not in the sense of lacking properties, but because for any non-trivial property P it happens to instantiate, the particular [= individual; J.R.] might conceivably have lacked P and still be the same thing (Tichý 2004, 717).

In 'Kripke on Necessity a Posteriori' Tichý dismissed Kripke's idea that a particular wooden table, let us say an individual T_1, cannot be woodless without changing its numerical identity, for, in order not to make the idea of empirical testing quite idle, we must be sure about the identity of T_1 (what T_1 is), independently of the result of the woodenness test, first. Tichý ends up his argumentation by the following claims:

> [An individual; J.R.] as such must be *bare*. Bare not in the sense of actually lacking properties, but in the sense that where P is any nontrivial property [an individual; J.R.] has, it is metaphysically *possible* for it to lack P and still be the same thing. Nontrivial properties belong to individuals not by metaphysical necessity, but as a matter of contingent fact (Tichý 1983, 241).

Finally, in the monograph 'The Foundation of Frege's Logic', Tichý expressed similar observations:

> Etna as such is a *bare* individual. It is bare not in the sense of actually lacking properties but in the sense that no non-trivial property it happens to instantiate is *constitutive* of it. Etna might conceivably lack any of these properties without thereby becoming its own numerically distinct individual. Only trivial properties, like its own numerical identity, are features that Etna cannot possibly lack (Tichý 1988, 210).

---

[5] Statements concerned with this were added in proof. Of course, their insertion may seem a bit artificially, thus we restrict ourselves only to those inserted. But the reader is invited to read section '1. Bare Individuals and Essentialism' (Tichý 2007) to confront with Tichý's opinions we are omitting here.

At this point, Tichý also explained:

> Frege fails to see that the aim of abstraction is not to *deprive* the individual of its properties, but conceptually separate it from them. To separate an individual conceptually from a property it in fact has, it is enough to *consider* a possible state of affairs in which the very same individual lacks that property. There is no need to make that state of affairs actual, as Frege seems to presuppose. An individual which is strictly distinguished from the properties it happens to instantiate is nothing but a pure individuator (Tichý 1988, 211).

Admittedly, some philosophers find these formulations not clear enough. Such a view, however, seems to be unjustified given that when we conceptually separate properties from an individual, the rest is simply a bare individual, a pure individuator (this claim is an *epistemological claim*). This is not to say that this individual is without properties (which would be a dubious ontological claim). Clearly, an individual cannot lack trivial properties (like its own self-identity), but it can lack non-trivial property it actually instantiates and can instantiate another non-trivial property. Contrary to this, essentialists insist on several non-trivial properties as indispensable: „being a philosopher" is an essential property of Socrates, „being born to George and Elisabeth Windsors" is an essential property of Elisabeth II, etc. In other words, they claim that there are several non-trivial properties which are constitutive of the real identity ('thisness') of the individuals in questions. And this is the thesis the antiessentialists deny and argue for conceiving of individuals as bare. Antiessentialists express an *ontological claim formulated as a conditional*, namely, that every property, if it is contingent and an individual instantiates it, can be lacked by that individual.

A sketchy comparison with other defenders of the idea of bare individuals might be helpful here. Firstly, Tichý did not strictly distinguish between individuals and particulars, but his most frequent term was 'individual'. But the topic of bare individuals is investigated nearly exclusively under the name 'bare particulars' (or even 'substrata theory' or recently 'thin particulars'). Secondly, Tichý never quoted any other theoretician discussing bare particulars. It seems, however, that he knew, perhaps by some indirect reference, Gustav Bergmann's proposal.[6] The

[6] This my long-term conviction just expressed is finally confirmed, *cf.* (Tichý 2007, 424; note that it is clear from my quotation from Bergmann's book that Tichý knew Bergmann's proposal indirectly – he attributes to Bergmann the use of the term 'individuator' but not the term 'bare').

difference between Tichý's doctrine and Bergmann's one is apparent from the following quotation:

> Bare particulars neither are nor have natures. Any two of them are not intrinsically but only numerically different. That is their bareness. ... A bare particular is a mere individuator.... It does nothing else (Bergmann 1967, 24 – 25).

The absence of inner nature in the discussed sense Tichý would not hesitate to confirm.[7] 'Numerical distinctiveness' as well as 'being a mere individuator' are both mentioned in Tichý's formulations. But there are two important dissimilarities. The theory of bare particulars is notoriously misinterpreted as a proposal according to which individuals do not have (certain kinds of) properties at all, or that they instantiate properties only on some occasions. But Tichý repeatedly and explicitly warned us against such a hasty conclusion: individuals do have properties already; individuals are not naked beings only occasionally clothed.[8] Nevertheless, empirical clothes as such are something that is not constitutive of them. In recent writings, J.P. Moreland's formulation of bare particulars seems to be the most enlightening with respect to the necessity to warn against the idea of particulars conceived as entirely without properties:

> bare particulars are simple and properties are linked or tied to them. ... A particular is called „bare", not because it comes without properties ... Since bare particulars are simples, there is no internal differentiation within them. ... bare particulars have a number of properties, e.g., being red, and they have some properties necessarily, e.g. particularity, in the sense that a bare particular can exist only if it has certain properties tied to it (Moreland 1998, 257 – 258).

Moreland's last note has more consequences than we may discuss in this paper. Let us therefore understand it only in the sense sympathetic to Tichý's opinion (thought, yet not explicitly formulated, *cf*. above quotations), namely that trivial properties are 'constitutive' for individuals. It does not mean, however, that this constitutiveness represents internal nature of individuals. But some theoreticians (such as Plantinga) conceive of certain trivial properties (such as „being identical with the indivi-

---

[7] It is also worth pointing out that individuals do not have other individuals as their inner content in Tichý's conception. Tichý is not a 'Polish logician', a mereologist.

[8] I am convinced that they are clothed enough all the time (in every possible world). Various arguments supporting this idea are offered in (Raclavský 2008a). It is argued there that this fact is perfectly compatible with the theory of bare individuals.

dual I_1") as so-called Leibnizian essence. Tichý did not refer to this idea in any of the passages discussing bare individuals. The following Tichý's claim, which seems to amount to an acceptance of Leibnizian essence, comes from his criticism of Plantinga's too wide concept of essence:

> the essence of any individual *x* is the rather dull and lackluster property of *x-identity* (Tichý 1972, 92).

It is not clear whether Tichý held this opinion also later. This doubt arises from the fact that several ideas advocated by Tichý in 1972 were later abandoned by him, as it is evident from Tichý (1976).

Nevertheless, independently of what has been said above we are completely sure that Tichý would not agree with the notion of individuals conceived (at least theoretically) as lacking trivial properties. It should be also noted that Tichý's transparent intensional logic is in fact not incorporating Leibnizian essentialism, for individuals are objects collected in the type ι, whereas properties, including „being identical with the individual I_1", are in the type we may sign now φ. The second main difference between Tichý's conception of bare individuals and those of other theoreticians is that Tichý never discussed the questions of individuation and recognition, which are typical topics in discussions of bare particulars. We need not deduce, for example, that he would simply agree with Edwin B. Allaire's classical defence (Allaire 1963) of bare individuals against Russellian Principle of Acquaintance by a reference to the difference between acquaintance with an individual and recognition of an individual. Acquaintance and recognition seems to be the same for Tichý – he did not mention any complications with regard to the recognition of the same individual based on its repeated occurrence. Tichý seems to rely on the idea that the individual occurring repeatedly is the same individual. Closely related difficulties with identification by spatio-temporal location (and the individuation possibly arising from this) are not discussed in Tichý's writings either.

Defining Tichý's notion of bare individuals

Before we examine Tichý's formulation of the conception of bare individuals, it is necessary to formulate it precisely first.[9] As soon as we extract

[9] The reader might be inclined to believe that Tichý's conception was firstly explicitly formulated and then rejected by Petr Kolář (*cf.*, e.g., Kolář 2000). Unfortunately, Kolář

the substantial ideas from Tichý's claims already quoted above, we obtain the following outline (the slashes indicate alternatives):

> *An individual is bare (a) not in the sense of actually lacking properties, but in the sense that (b) where P is any non-trivial property the individual instantiates (1983 version: has), (c) it is metaphysically possible for it to (or: it might conceivably) lack P; and (d) still be the same thing/(e) without thereby becoming its own numerically distinct individual/(f) and only trivial properties are such that the individual cannot possibly lack.*

It seems that we have several options how to define bare individuals in accordance with Tichý's proposal. Part (a) says how not to construe bare individuals thus it is not already a part of the definition.[10] Parts (b) (which consists of conjuction) and (c) are closely related as an antecedent and a consequent of a material conditional (note that properties are quantified not in the consequent as critics of bare individuals wrongly assume, but in part (b)). Parts (d)-(f) can be viewed as additions to the consequent or to the whole conditional. I suggest to choose only (f) to be added by conjunction, so completing (b)-(c) + (f). Given that (d) "being the same" means obviously the same as "being self-identical", if it occurs in the consequent of the implication, the implication will be simply true (analogously for (e)). On the other hand, (f) would, at least theoretically, cause falsity of the whole definiens especially, when we add it by means of conjunction to the whole conditional. Hence, I suggest $((b)\rightarrow(c))\wedge(f)$ as a basis for our further explication.

One of the most important terms in Tichý's formulation is 'can lack'. It would be convenient, however, to realize first the sense of the predicate 'instantiate', whose negation can be reasonably applied to an individual-property couple also when the respective property is undefined for a given world. To claim then that an individual instantiates such property in that world would be false on such occasion, not without a truth value. Hence to claim that the individual in question does not in-

---

formalized wholly different conceptions (not of bare, but rather of 'naked' individuals) and he completely failed to examine Tichý's one. Kolář's alleged refutation is reviewed and criticized in detail by Raclavský (2007b).

[10] Note that individuals lacking non-trivial as well as trivial properties (both covered by the term 'properties') are really absurd. Pavel Cmorej (2006) criticized (and I assume that rightly) also other conceptions of individuals lacking all properties of certain kind. Yet none of those conceptions corresponds to Tichý's conception of bare individuals (what Cmorej attacked were rather conceptions of 'naked' individuals).

stantiate that property would be true. Thus in contrast to the possibly truth-valueless statement like 'I_1 is an F' (as an illustration, consider the truth-value-lacking claim 'I_1 is a brother of the king of France'), we suppose a 'total' way of attributing properties which ensures that our statements are definitely true or false (but not truth-value-less).

We will define the predicate 'instantiate' by means of the 'totalizing' predicate 'being true' (formal definitions are to be found in the appendix):[11]

> an individual x *instantiates* a property $f =_{df}$ it is true that an individual x is an f

Then it would be natural to explicate the predicate 'lack' as follows:

> an individual x *lacks* a property $f =_{df}$ an individual x does not instantiate f

Let us thus assume the concept CanLack that may be expressed also by the phrase 'being possible for x to lack f' (*cf.* with Tichý's 'metaphysically possible'), whereas being possible is explained by the existence of at least one possible world in which the lacking obtains. Note that in the natural sense CanLack *presupposes* that it is possible for an individual to have the property in question which may loose.[12] For illustration, consi-der the property „being an individual identical with I_2" – it is obvious that the individual I_1 cannot possibly have this property, thus it cannot be true that I_1 can lack it. Thus:

> an individual x *can lack* a property $f =_{df}$ there exists a possible world w' such that x is an f in w' and there exists a possible world w'' such that x lacks f in w''

(The attribute „cannot lack" can be obtained simply by mere 'negation' of „can lack".)

---

[11] It is evident from Tichý (1976), especially section 30, that we are presenting here just the concept Tichý had in mind. He considered here a strictly total (world-dependent) relation H between ξ-objects and ξ-properties which is denoted in "the English language [...] by the verb 'instantiate' ([...], 'exemplify', 'partake of')." . Nevertheless, the identification of „instantiate" with „it is true that an individual x is an f" cannot be documented (most probably for the reason that „true" is introduced much further in Tichý's book (1976) for it is a propositional attribute and these were investigated further on than properties of individuals).

[12] On the contrary assumption, we would immediately fall in the paradox exactly similar to The Horned Man.

Now according to the suggested ((b)→(c))∧(f) reading of Tichý's formulation of bare individuals we state just:

> being a *bare*^{T} individual x $=_{df}$ being an individual x such that for every property f, if f is a non-trivial property and x has f, then x can lack f; and for every property g, if g is trivial, then x cannot lack g

whereas 'x has f' comes from the 1983 version. To satisfy the more technical reader we shall introduce formal definitions of concepts related to Tichý's proposal directly into the main text, i.e. not in the appendix (where notational agreements and shortcuts are explained; we will omit the outermost brackets on the right side of the formal definition):

$$[\mathrm{Bare}^{T}_{w}\ x] \equiv^{(o\iota)\omega} [\forall.\lambda f [\ [\ [\mathrm{NonTrivial}_{w}\ f] \wedge [f_{w}\ x]\ ] \rightarrow [\mathrm{CanLack}_{w}\ x\ f]\ ]] \\ \wedge [\forall.\lambda g\ [\ [\mathrm{Trivial}_{w}\ g] \rightarrow \neg[\mathrm{CanLack}_{w}\ x\ g]\ ]] \qquad (\lambda w.\lambda x.)$$

However, the 1983 variant has one problem in the first part of the main conjunction if we consider a somewhat uncharitable understanding of 'to have the property f'; in the definiens of Bare^{T}, we identified it with $[f_{w}\ x]$. Then the part (b), formalized as $[\ [\mathrm{NonTrivial}_{w}\ f] \wedge [f_{w}\ x]\ ]$, returns a truth-value only when the property constructed by f is total. But there are many properties which are partial and a certain individual actually lacks them. Let us suppose, for example, the property denoted by the expression 'being a sibling of the king of France'; this property is undefined for the actual world (and many others), thus $[f_{w}\ x]$ does not return any truth-value. Consequently, the universal quantifier assigns to this property the truth-value F. So the class of individuals which are bare would be empty because any individual does not actually instantiate certain non-trivial and dispensable properties.

In order to avoid the consequence just mentioned, we have two options: either to understand 'to have the property F' as 'instantiates the property F' in our sense – thus adopting more frequent variants of Tichý's formulation, or to read 'the property F the individual has' in the sense of the 1983 version as a stylistic hint at the part (a), i.e. 'not actually lacking the property F'. Both variants are in fact equivalent. Let us start with the former one because it is the most probable interpretation of Tichý:

> being a *bare*$^{T'}$ individual x $=_{df}$ being an individual x such that for every property f, if f is a non-trivial property and x instantiates f, then x can lack f; and for every property g, if g is trivial, then x cannot lack g

$$[\mathrm{Bare}^{\mathrm{T}'}{}_w\, x] \equiv^{(o\iota)\omega} [\forall.\lambda f [\, [\, [\mathrm{NonTrivial}_w f] \wedge [\mathrm{Instantiate}_w\, x\, f]\,] \rightarrow [\mathrm{CanLack}_w\, x\, f]\,]] \wedge [\forall.\lambda g [\, [\mathrm{Trivial}_w\, g] \rightarrow \neg[\mathrm{CanLack}_w\, x\, g]\,]]$$

Partial non-trivial properties which are not instantiated by individuals cause only falsity of the (first) antecedent (for it is not true that the value of x is in the extension of f), thus the whole implication would be true and not without a truth-value; so the universal quantifier will return T to such properties.[13] The latter option reads:

being a *bare*$^{\mathrm{T}''}$ individual x $=_{df}$ being an individual x such that for every property f, if f is a non-trivial property and x does not actually lack f, then x can lack f; and for every property g, if g is trivial, then x cannot lack g

$$[\mathrm{Bare}^{\mathrm{T}''}{}_w\, x] \equiv^{(o\iota)\omega} [\forall.\lambda f [\, [\, [\mathrm{NonTrivial}_w f] \wedge \neg[[\mathrm{Lack}\, [\mathrm{Actual}\, w]]\, x\, f]\,] \rightarrow [\mathrm{CanLack}_w\, x\, f]\,]] \wedge [\forall.\lambda g [\, [\mathrm{Trivial}_w\, g] \rightarrow \neg[\mathrm{CanLack}_w\, x\, g]\,]]$$

A simple equivalence of 'not lack' with 'instantiate' (thanks to our definitions) is obvious. But there is also the equivalence of 'individual is /any suitable predicate/ in actual world' with 'individual is /any suitable predicate/' due to the fact repeatedly claimed by Tichý (*cf.*, for example, Tichý 2004, 218) because 'actual' (or 'actual world') denotes an identity function assigning worlds to worlds (see the appendix). Bare$^{\mathrm{T}''}$ is thus equivalent to Bare$^{\mathrm{T}'}$.

There is, however, one serious shortcoming in any of Tichý's formulations of the conceptions of bare individuals, which was (indirectly) detected by Pavel Cmorej (2001).[14] Consider the following example of the so-called partly essential properties, whose existence was referred to, for instance, by Cmorej: „to have the same height as I_1". The property is non-trivial and I_1 has it, the antecedent of the first implication in the definition of Bare ($^{\mathrm{T}}$, $^{\mathrm{T}'}$, $^{\mathrm{T}''}$) is true. But there is no world in which it would be possible for I_1 not to have it. Hence, the first consequent in the definition of Bare ($^{\mathrm{T}}$, $^{\mathrm{T}'}$, $^{\mathrm{T}''}$) is undoubtedly false and the implication would then return the truth-value F for such property. Therefore, I_1 is not placed into the

[13] It is not necessary to strengthen the first antecedent by help of 'there exists an f' to which f is identical and x instantiates this f'' ($[\forall.\lambda f [\, [\exists.\lambda f' [\, [f'{=}f] \wedge [f'_w\, x]\,] \wedge [\mathrm{NonTrivial}_w f]\,] \rightarrow [\mathrm{CanLack}_w\, x\, f]\,]] \wedge \ldots$).

[14] In fact, Cmorej did not attack directly Tichý's formulations of the conception of bare individuals. The objection against Tichý's formulation is first explicitly stated in the present text.

extension of the respective property defined as „being bare". Then consider the property „to have the same height as I_2" and you will conclude that I_2 would not be in the extension of the defined property either. Analogous considerations apply to other individuals and partly essential properties, too. It follows that the extension of the property specified by any of Tichý's definitions is empty. Moreover, it is easy to see that it will be empty in any possible world. Any of *Tichý's definitions defines* (the concept of) *trivial empty property*, i.e. a property whose invariant extension is an empty class. Thus there is no world in which there exists at least one individual possessing such property. Consequently, *no individual is bare*. Therefore, *each of Tichý's formulations of the conception of bare individuals is wrong*.[15]

A reformulation of Tichý's conception of bare individuals

Although Tichý's definition(s) of bare individuals has to be rejected, it does not follow that one must give up the whole idea of antiessentialism (bare individuals) as it was originally intended especially against essentialists like Plantinga or Kripke (who do not take into account partly essential properties). As a follower of Tichý's antiessentialism, I am obliged to fix Tichý's original formulation in such a way that those non-trivial properties with extensions changing dependently on worlds and having a stable non-empty subclass in their extensions – i.e. partly essential properties – should be covered in a 'realistic' definition of the conception of bare individuals. In order to do so, we need to come up with a substantially richer classification of properties than was Tichý's original one.

We will in fact follow Cmorej (1996, 2001) who first proposed certain definitions of properties essential for (a specific individual), essential properties, purely essential properties, partly essential properties, purely empirical (contingent) properties within the framework of Tichý's transparent intensional logic. We will frequently use the definitions (or their equivalents) rigorously and formally exposed in (Raclavský 2007). In this study, properties accidental for (a specific individual), accidental properties, and void properties were also precisely defined. The *quadruplet di-*

[15] Tichý may be partly excused for assuming that he tacitly thought of so-called primary properties which are denoted by single expressions. On the other hand, so-called derived properties are denoted by compound expressions. No doubt, partly essential properties (*cf.* the expression 'having the same height as I_1') are derived. (This does not mean, however, that there are, within Tichý's system, only primary properties.)

viding properties into properties: (a) *purely empirical*, (b) *partly essential*, (c) *purely essential*, (d) *trivially void* (Cmorej's triplet is now completed by the last category of the so-called trivially void properties). Whereas (a) and (b) are non-trivial (empirical, contingent), (c) and (d) are trivial (non-empirical); on the other hand, (b) and (c) are essential, but (a) and (d) are non-essential. (See Raclavský 2007 for details.)

One of the most important concepts for us is *EssentialFor*:

> being a property *f essential for* the individual I_1 $=_{df}$ being a property such that in every possible world w' the individual I_1 is in the extension of *f* in w'

This definition is evidently equivalent to:

> $=_{df}$ being a property such that in every world w' the individual I_1 instantiates *f* (in w')[16]

By means of existential generalization we get the concept *Essential*:

> being an *essential* property *f* $=_{df}$ being a property such that there exists an individual *x* for which *f* is essential in every world w'

which is equivalent to:[17]

> $=_{df}$ being a property such that there exists an individual *x* which cannot lack *f*
> $=_{df}$ being a property such that there exists an individual *x* which instantiates *f* in every world w'

Further we will take an advantage from the definition of *AccidentalFor*:

> being a property *f accidental for* the individual I_1 $=_{df}$ being a property such that there exists world w' such that I_1 is in the extension of *f* and there exists world w'' in which I_1 lacks *f*

---

[16] The equivalence to 'being a property such that the individual I_1 cannot lack *f*' does not hold; only the implication is correct.

[17] Every definition of a certain concept is relative to a given conceptual system. It is sometimes thought that within a given conceptual system only one definition of certain concept is reasonably conceivable. Since we do not offer a specific conceptual system, we suggest various definitions each of them relative to different conceptual system. On the other hand, when conceptual system is rich enough, then there are, no doubt, different possibilities how to define one concept. Likewise, the opinions on the matter whether there are concepts defined (by means of definition) or objects determined by concepts differ (we prefer the first view though).

By existential generalization we obtain the concept *Accidental*:

being an *accidental* property $f =_{df}$ being a property such that there exists an individual x such that there exists world w' in which x is an f and there exists world w'' in which x lacks f

Now *essential* properties are divided into *purely essential* and *partly essential* properties:[18]

being a *purely essential* property $f =_{df}$ being a property which is trivial and essential (*)
$=_{df}$ being a property f which is essential and not accidental
being a *partly essential* property $f =_{df}$ being a property which is non-trivial and essential (*)
$=_{df}$ being a property f which is essential and accidental

However, *non-essential* properties can be divided into *purely empirical* and *trivially void* properties (these should not be confused with void properties in general because among void properties there are also non-trivially void properties which fall into the category of purely empirical properties):

being *purely empirical* property $f =_{df}$ being a property which is not trivial and not partly essential (*)
$=_{df}$ being a property which is non-essential and accidental
being a *void* property $f =_{df}$ being a property which is not essential and not accidental
being a *trivially void* property $f =_{df}$ being a property which is void and trivial
being a *nontrivially void* property $f =_{df}$ being a property which is void and not trivial

We have said that to avoid Cmorejian objection we should reformulate Tichý's original proposal. The easiest way is to adopt Bare$^{T''}$ and simply replace 'non-trivial' just by 'accidental for' ('^{R}' for 'real'):

being a *bare*^{R} individual $x =_{df}$ being an individual x such that for every property f, if f is accidental for x and x instantiates f, then x can lack f; and for every property g, if g is trivial, then x cannot lack g

[18] The formulations below ending by '(*)' are a bit imprecise (the reason, which is rather 'technical', is explained in Raclavský 2008a; the same inaccurateness affects also our formalisations of T, T', T'').

$$[\text{Bare}^{\text{R}}_{w}\ x] \equiv^{(o\iota)\omega} [\forall.\lambda f [\ [\ [\text{AccidentalFor}_{w}\ f\ x] \wedge [\text{Instantiate}_{w}\ x\ f]\ ] \rightarrow [\text{CanLack}_{w}\ x\ f]\ ]] \wedge [\forall.\lambda g\ [\ [\text{Trivial}_{w}\ g] \rightarrow \neg[\text{CanLack}_{w}\ x\ g]\ ]]$$

The property constructed by Bare^{R} is the only *trivial universal property*. This is purely essential, trivial, total and having non-empty (moreover maximal) extension. (Note that our key 'trick' lies in a certain abstraction from a particular individual for which the property *f* should be accidental, thus allowing any value of *x*.)

Let us now investigate the obvious equivalent of the definiens, mainly the equivalence of its first implication containing more basic concepts (the construction is α-reduced):

$$[\forall.\lambda f [\ [\ [\exists.\lambda w' [\ [f_{w'}\ x] \wedge [\exists.\lambda w' [\text{Lack}_{w'}\ x\ f]]\ ]] \wedge [\text{True}_{w}\ [\lambda w' [f_{w'}\ x]]]\ ] (\ [(.\lambda w' [\ [f_{w'}\ x]\ (\ [(.\lambda w' [\text{Lack}_{w'}\ x\ f]]\ ]]\ \ ]]$$

It is apparent that the only difference between the antecedent and the consequent of this implication lies in the allowing of that property *f* which may be perhaps instantiated by an individual *x*, i.e. the 'instantiation condition' $[\text{True}_{w}\ [\lambda w' [f_{w'}\ x]]]$. This makes the only exception from the tautological scheme $(c \rightarrow c)$. One might perhaps object that the definition is 'tautologically void' after all. It should be realized, however, that „Bare^{R}" was intended as a property possessed by each individual (the trivial universal property) and this naturally causes the tautological character of its (conceptual) definition.

The obvious equivalent of Bare^{R} was just indicated. Let us examine another formulation of Bare that comes from $\text{Bare}^{\text{T}''}$ by the simple replacement of 'not essential for' for 'not trivial':

being a *bare*$^{\text{R}'}$ individual $x =_{df}$ being an individual *x* such that for every property *f*, if *f* is not essential for *x* and *x* instantiates *f*, then *x* can lack *f*; and for every property *g*, if *g* is trivial, then *x* cannot lack *g*

$$[\text{Bare}^{\text{R}'}_{w}\ x] \equiv^{(o\iota)\omega} [\forall.\lambda f [\ [\ \neg[\text{EssentialFor}_{w}\ f\ x] \wedge [\text{Instantiate}_{w}\ x\ f]\ ] \rightarrow [\text{CanLack}_{w}\ x\ f]\ ]] \wedge [\forall.\lambda g\ [\ [\text{Trivial}_{w}\ g] \rightarrow \neg[\text{CanLack}_{w}\ x\ g]\ ]]$$

The difference between Bare^{R} and $\text{Bare}^{\text{R}'}$ is substantial. It is so because properties accidental for a certain individual are purely empirical or partly essential, whereas properties non-essential for a certain individual are purely empirical or partly essential or trivially void, or − and this is important − purely essential (when they are essential for another individual). When we have some purely essential property, for instance the trivial singular property with the extension containing I_2, and the indi-

vidual I_1, then the antecedent (of the first implication) is false because I_1 does not instantiate this property; the consequent is false too, so the implication will be true. Note, that $\neg$[EssentialFor$_w$ f x] can be converted to $\neg[\forall.\lambda w' [f_{w'} x]]$, but then to $[\exists.\lambda w' \neg[\text{True}_{w'} [\lambda w'' [f_{w''} x]]]]$;[19] this also shows the difference between Bare^R and Bare$^{R'}$. Despite the difference between Bare^R and Bare$^{R'}$, both constructions construct the very same property of individuals, which means that Bare^R and Bare$^{R'}$ are equivalent.

Of course, we may change also the second conjunct of the definition. Since testing of all trivial properties, including trivially void properties, is, though harmless, somewhat redundant:

being a *bare*$^{R''}$ individual $x =_{df}$ being an individual x such that for every property f, if f is not essential for x and x instantiates f, then x can lack f; and for every property g, if g is essential for x, then x cannot lack g

$[\text{Bare}^{R''}{}_w\, x] \equiv^{(o\iota)\omega} [\forall.\lambda f [[\neg[\text{EssentialFor}_w\, f\, x] \wedge [\text{Instantiate}_w\, x\, f]]$
([CanLackw x f]]] ([(.λg [[EssentialForw g x] (([CanLackw x g]]]

It is easy to conclude that Bare$^{R''}$ constructs, again, the trivial universal property, thus Bare$^{R''}$ is equivalent to Bare^R and Bare$^{R'}$. Finally, we can combine definitions of Bare^R and Bare$^{R''}$ in the following way:

being a *bare*$^{R'''}$ individual $x =_{df}$ being an individual x such that for every property f, if f is accidental for x and x instantiates f, then x can lack f; and for every property g, if g is essential for x, then x cannot lack g

$[\text{Bare}^{R'''}{}_w\, x] \equiv^{(o\iota)\omega} [\forall.\lambda f [[[\text{AccidentalFor}_w\, f\, x] \wedge [\text{Instantiate}_w\, x\, f]]$
$\rightarrow [\text{CanLack}_w\, x\, f]]] \wedge [\forall.\lambda g [[\text{EssentialFor}_w\, g\, x] \rightarrow \neg[\text{CanLack}_w\, x\, g]]]$[20]

---

[19] Working with partial functions, we cannot uncritically apply the classical formulations of De Morgan's Laws for the interchange of quantifiers.

[20] There is one remarkable feature of Tichý's formulations which was retained in our new definitions. Suppose the non-trivial property alternating the empty class and the singleton $\{I_1\}$ as its extensions and an individual I_1. If the antecedent (b) were formulated only as 'if f is non-trivial', then the implication can be false (given the antecedent's being true while consequent's being false). Tichý's condition 'and x instantiates this f' in (b) precludes such result. Since our definitions of Bare^R, Bare$^{R'''}$ count with the fact that the properties accidental for an individual are such that there is at least one possible world in which the individual has such property(-ies), 'and x instantiates this f' can be omitted in them (we obtain then Bare^{R*}, Bare$^{R*''}$). On the other hand, Bare$^{R'}$ and Bare$^{R''}$ cannot be simplified this way (in the definition of non-essential properties for certain individuals we do not presuppose possible instantiation of those properties by them).

One might object that according to such definitions of bare individuals, individuals instantiating properties accidental (or non-essential) for them may lack these properties and they cannot lack properties essential for them (of course, not all of both kinds in the same world). Such a claim seems to be then tantamount to the claims of old metaphysicians and modern essentialists, one would conclude. In certain sense it is true – nevertheless, in another important sense it is not. At first, it would be undeniably odd to entirely contravene certain good ideas of other thinkers. However, we say 'No' to certain properties considered by classical or modern essentialists as indispensable (recall, for instance, the case of T_1's being wooden). The class of properties essential for certain individual construed in our sense is rather narrower than the class of essential properties determined by essentialists. As soon as there exists a possible world in which an individual lacks a property considered by classical essentialist as essential for it, then this property is not, according to our conception, really essential for it.[21]

Katedra filozofie
Filozofická fakulta
Masarykova univerzita
Arne Nováka 1
602 00 Brno
raclavsk@phil.muni.cz

REFERENCES

ALLAIRE, E.B. (1963): Bare Particulars. *Philosophical Studies* 14, Nos. 1 – 2, 1 – 8.

BERGMANN, G. (1967): *Realism: A Critique of Brentano and Meinong*. Madison: University of Wisconsin Press.

CMOREJ, P. (1996): Empirické esenciálne vlastnosti. *Organon F* 3, No. 3, 239 – 261.[22]

CMOREJ, P. (2001): Esencializmus verzus antiesencializmus. In: *Na pomedzí logiky a filozofie*. Bratislava: Veda, 91 – 113.[23]

CMOREJ, P. (2006): Holé indivíduá a predikácia. In: M. Zouhar (ed.): *Jazyk z pohľadu sémantiky, pragmatiky a filozofie vedy*. Bratislava: Veda, 137 – 161.

[21] The author of this article is supported by the grant of GAČR no. 401/07/P280.

[22] Reprinted in Cmorej, P. (2001): *Na pomedzí logiky a filozofie*. Bratislava: Veda, 125 – 149.

[23] Originally published as Cmorej, P. (1988): Esencializmus versus antiesencializmus. *Philosophica* 26, Univerzita Komenského: Bratislava, 163 – 202.

KOLÁŘ, P. (2000): Clothing Bare Individuals. In: O. Majer (ed.): *Topics in Conceptual Analysis and Modelling*. Praha: Filosofia, 126 – 139.

MORELAND, J.P. (1998): Theories of Individuation: A Reconsideration of Bare Particulars. *Pacific Philosophical Quaterly* 79, No. 3, 251 – 263.

RACLAVSKÝ, J. (2007): Defining Basic Kinds of Properties. In: Marvan, T. – Zouhar, M. (eds.): *The World of Language and the World beyond Language (A Festchschrift for Pavel Cmorej)*. Bratislava: Veda, 69 – 107.

RACLAVSKÝ, J. (2007b): Against Kolář's Criticism of Tichý's Bare Individuals. *Pro-Fil* 8, No. 2, http://profil.muni.cz/02_2007/raclavsky_against_kolars_criticism.pdf.

RACLAVSKÝ, J. (2008): On the Definitions of Basic Kinds of Properties. *Organon F* 15, No. 1, 80 – 83.

RACLAVSKÝ, J. (2008a): Bare Individuals are not without Properties. To appear in the proceedings from the conference of the Czech and Slovak analytic philosophy held in Stará Lesná, 2007.

TICHÝ, P. (1972): Plantinga on Essence: A Few Questions. *The Philosophical Review* 81, No. 1, 82 – 93.[24]

TICHÝ, P. (1976): *Introduction to Intensional Logic*. Unpublished book manuscript.

TICHÝ, P. (1983): Kripke on Necessity a posteriori. *Philosophical Studies* 43, No. 2, 241 – 255.

TICHÝ, P. (1988): *The Foundations of Frege´s Logic*. Berlin – New York: Walter de Gruyter.

TICHÝ, P. (1994): Jednotliviny a ich roly. *Organon F* 1, No. 1, 29 – 42, No. 2, 123 – 132, No. 3, 208 – 224, No. 4, 328 – 333.

TICHÝ, P. (2004): *Pavel Tichý's Collected Papers in Logic and Philosophy*. V. Svoboda, B. Jespersen, C. Cheyne (eds.). Dunedin: Otago UP, Praha: Filosofia.

TICHÝ, P. (2007): On Describing. *Organon F* 14, No. 4, 423 – 469.

APPENDIX

In the shortcut notation of transparent intensional logic (see Tichý 1988, Tichý 2004), which is introduced and explained in (Raclavský 2007), formal definitions of various concepts mentioned in our text are offered[25] (what is omitted are mainly the signs for trivializations and indication of temporal dependence[26]).

[24] Tichý's (sometimes co-authored) published papers are reprinted (if they were not originally written in English, they are translated) in Tichý (2004).

[25] Most definitions already appeared in Raclavský (2007) (see this text for more related details).

[26] The temporal versions of our definitions are easy to obtain. It would be sufficient just to write '$\lambda w.\lambda t$' instead of 'λw' and '$...{}_{wt}...$' instead of '$...{}_{w}...$' (using here the convention

Tichý's logic is a higher-order intensional logic handling total and partial functions. The functions are built upon a base consisting in collections of individuals, ι, truth-values, o (including only T and F), possible-worlds, ω, and real numbers/time-moments, τ. Functions and other objects are constructed by so-called constructions, abstract and structured procedures (in a nutshell, they are objectually viewed λ-terms). Variable x (or y) constructs ι-objects (individuals), variable w (or w', w'') constructs ω-objects (possible worlds), variable s constructs (οι)-objects (classes of individuals), variable f (or g) constructs $(o\iota)_\omega$-objects (properties of individuals), variable o constructs o-objects, variable p constructs o_ω-objects (propositions). The type $(o\iota)_\omega$ will be written briefly as φ, the type o_ω as π. Compositions $[X\ w]$ will be written as X_w.

The purpose of each such definition is to specify which object would be constructed by the construction on the left side. Both constructions related by the operator $\equiv^\xi$ construct, dependently on any valuation, the very same object (if they construct with respect to the particular valuation anything at all). Constructions on both sides are open constructions; for easier understanding we will indicate in the round brackets the missing „binding string" such as '$\lambda w.\lambda x f.$', which may close each of the constructions.[27] In definitions following some previous definitions, we will use η-reduced (even η-normalized) forms of constructions from the previous definitions (schematically, $\lambda x y\ [X\ x y]$ is η-reducible to X). Let ξ or ξ_i be an arbitrary type. The type ξ written in '$\equiv^\xi$' is a type of object constructed by the construction (on each side) after its closure by the respective binding item. Note, however, that the equality $\equiv^\xi$ does not relate just ξ-objects but certain ξ_i-objects which are constructed by open constructions on both sides; thus the type of $\equiv^\xi$ is in fact $(o\xi_i\xi_i)$. Nevertheless, the inscription '$\equiv^\xi$' contains information about which type the type ξ_i actually is. When ξ is, for instance, $(o(o\iota)\varphi)_\omega$ and we read '$\lambda w.\lambda s f.$' near the definition, then ξ_i is $(o(o\iota)\varphi)_\omega$ minus ω (due to '$\lambda w.$') and minus $(o\iota)\varphi$ (due to '$\lambda x f.$'), thus ξ_i is just o (i.e. '$\equiv^{(o(o\iota)\varphi)\omega}$' denotes here an equality of type (ooo)). If not indicated in a footnote otherwise, all defined intensions are total functions. If these intensions are trivial (constant), we put the variable w (in the left part of the definition) into vertical bars.

What follows are a few auxiliary concepts (sing is singularization, 'descriptive operator'):

$$[\mathrm{True}_w\ p] \equiv^{(o\pi)\omega} [\exists.\lambda o\ [\ [o = p_w] \wedge [o = \mathrm{T}]\ ]] \qquad (\lambda w.\lambda p.)$$

---

that $[[X\ w]\ t]$ is abbreviated as X_{wt}). Of course, when it is originally used, for example, the variable w'', then we should use also the variable t''.

27 If the constructions in the immediately following definition should be closed by the same binding string, we do not repeat this indication. Of course, the reader should complete the proper record of construction in his/her mind by supplying the respective right brackets in the right places.

$[\text{ExtensionOf}_w\ f] \equiv^{((o\iota)\varphi)\omega} [\text{sing}.\lambda s\ [s = f_w]]$ (i.e. $\equiv^{((o\iota)\varphi)\omega} f_w$) $(\lambda w.\lambda f.)$[28]

First four kinds of properties of individuals known already to Tichý (variables are sometimes superfluously α-renamed for easier understanding; note that w and w' may be assigned by the same possible world):

$[\text{Total}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\forall.\lambda w'[\exists.\lambda s\ [\ s = [\text{ExtensionOf}_{w'}\ f]\ ]]]$ $(\lambda w.\lambda f.)$
$[\text{Partial}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\exists.\lambda w' \neg[\exists.\lambda s\ [s = [\text{ExtensionOf}_{w'}\ f]\ ]]]$
$[\text{Trivial}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\forall.\lambda w'\ [\text{True}_{w'}\ [\lambda w''[\ [\text{ExtensionOf}_{w''}\ f] =$
$[\text{ExtensionOf}_w\ f]\ ]]] \vee [\neg\exists.\lambda w'''\ [\exists.\lambda s\ [s = [\text{ExtensionOf}_{w'''}\ f]]]]\ ]$
$[\text{NonTrivial}_{|w|}\ f] \equiv^{(o\varphi)\omega} \neg[\text{Trivial}_w\ f]$[29]

Four special sorts of trivial properties of individuals:

$[\text{Empty}^{\text{Tr}^{\varnothing}}{}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\forall.\lambda w'\ [\ [\text{ExtensionOf}_{w'}\ f] = \varnothing]]$ $(\lambda w.\lambda f.)$
$[\text{Undefined}^{\text{Tr}}{}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\forall.\lambda w'\ \neg[\exists.\lambda s\ [s = [\text{ExtensionOf}_{w'}\ f]\ ]]]$
$[\text{Universal}^{\text{Tr}}{}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\forall.\lambda w'\ [\ [\text{ExtensionOf}_{w'}\ f] = \lambda x\text{T}\ ]]$[30]
$[\text{Singular}^{\text{Tr}}{}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\ [\forall.\lambda w'\ [\exists.\lambda x\ [\ [[\text{ExtensionOf}_{w'}\ f]\ x] \wedge$
$[\forall.\lambda y\ [\ [[\text{ExtensionOf}_{w'}\ f]\ y] \rightarrow [y{=}x]\ ]\ ]]]\ \wedge [\text{Trivial}_w\ f]\ ]$

Definitions of concepts needed for the definitions of Tichý's conception of bare individuals:

$[\text{Instantiate}_w\ x\ f] \equiv^{(o\iota\varphi)\omega} [\text{True}_w\ [\lambda w'\ [f_{w'}\ x]]]$ $(\lambda w.\lambda x f.)$
$[\text{Lack}_w\ x\ f] \equiv^{(o\iota\varphi)\omega} \neg[\text{Instantiate}_w\ x\ f]$
$[\text{CanLack}_{|w|}\ x\ f] \equiv^{(o\iota\varphi)\omega} [\ [\exists.\lambda w'\ [f_{w'}\ x]] \wedge [\exists.\lambda w''\ [\text{Lack}_{w''}\ x\ f]]\ ]$
$[\text{CanInstantiate}_{|w|}\ x\ f] \equiv^{(o\iota\varphi)\omega} [\exists.\lambda w'\ [f_{w'}\ x]]$

Now let the variable v-construct mappings assigning worlds to worlds (and 'sing' a singularization function):

$[\text{Actual}\ w] \equiv^{(\omega\omega)} [\text{sing}.\lambda v\ [\forall.\lambda w'\ [\ [\text{ExtensionOf}^{\omega}{}_{w'}\ v] = w\ ]]]_w$
(i.e. $\equiv^{(\omega\omega)} w$) $(\lambda w.)$

Thus for our case we have:

$[\ [\text{Lack}\ [\text{Actual}\ w]]\ x\ f] \equiv^{(o\iota\varphi)\omega} [\text{Lack}_w\ x\ f]$ $(\lambda w.\lambda x f.)$
$\neg[\ [\text{Lack}\ [\text{Actual}\ w]]\ x\ f] \equiv^{(o\iota\varphi)\omega} [\text{Instantiate}_w\ x\ f]$

---

28 The construction $[\text{ExtensionOf}_w\ f]$ may be improper (i.e. does not construct anything at all) if the value of f is not defined in a given particular world; „extension of" is a partial mapping.

29 When certain property is total, then its correct complement, the property *complementary to* it, is constructible by means of negation placed properly within the original concept-construction of the former one.

30 The construction $\lambda x\text{T}$ constructs the class of all individuals.

Definitions of further kinds of properties:

$[\text{EssentialFor}_{|w|}\ f\ \text{I}_1] \equiv^{(o\varphi)\omega} [\forall.\lambda w'\ [[\text{ExtensionOf}_{w'}\ f]\ \text{I}_1]]$ $(\lambda w.\lambda f.)$

$[\text{Essential}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\exists.\lambda x\ [\text{EssentialFor}_{w}\ f\ x]]$

$(\equiv^{(o\varphi)\omega} [\exists.\lambda x\ [\forall.\lambda w'\ [\text{Instantiate}_{w'}\ x\ f]]])$

$[\text{AccidentalFor}_{|w|}\ f\ \text{I}_1] \equiv^{(o\varphi)\omega}$

$[\exists.\lambda w'\ [\ [[\text{ExtensionOf}_{w'}\ f]\ \text{I}_1] \wedge [\exists.\lambda w''\ [\text{Lack}_{w''}\ \text{I}_1\ f]]\ ]]$

$[\text{Accidental}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\exists.\lambda x\ [\exists.\lambda w'\ [\ [f_{w'}\ x] \wedge [\exists.\lambda w''\ [\text{Lack}_{w''}\ x\ f]]\ ]]]$

$[\text{PurelyEssential}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\ [\text{Essential}_{w}\ f] \wedge \neg[\text{Accidental}_{w}\ f]]$

$[\text{PartlyEssential}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\ [\text{Essential}_{w}\ f]\ \wedge [\text{Accidental}_{w}\ f]\ ]$

$[\text{PurelyEmpirical}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\ \neg[\text{Essential}_{w}\ f] \wedge [\text{Accidental}_{w}\ f]\ ]$

$[\text{Void}_{|w|}\ f] \equiv^{(o\varphi)\omega} [\ \neg[\text{Essential}_{w}\ f]\ \wedge \neg[\text{Accidental}_{w}\ f]\ ]$

$[\text{TriviallyVoid}_{|w|}\ f]\ \equiv^{(o\varphi)\omega} [\ [\text{Void}_{w}\ f] \wedge [\text{Trivial}_{w}\ f]\ ]$

$[\text{NonTriviallyVoid}_{|w|}\ f]\ \equiv^{(o\varphi)\omega} [\ [\text{Void}_{w}\ f] \wedge [\text{NonTrivial}_{w}\ f]\ ]$