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Common Source of the Paradoxes of Inference and Analysis

KAREL ŠEBELA

Department of Philosophy. Faculty of Arts. Palacký University in Olomouc
Křížkovského 12. 771 80 Olomouc. Czech Republic
karel.sebela@upol.cz

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ABSTRACT: The paper deals with the paradoxes of inference and analysis. It attempts to show what is specific about these paradoxes. They have got a lot in common. Often, they are not considered paradoxes in the strict sense at all. Moreover, they both raise the same problem: How can the requirements of correctness and informativeness be both met for inference and for conceptual analysis? The strategies developed to address the problem are similar for both cases. In the paper, I claim that the paradoxes have common origins. This claim is supported by comparing different strategies adopted to resolve the problem. Regarding their origins, both paradoxes share the epistemological framework that is grounded in Aristotle's theory of science. This is related to the problem of implicit knowledge, which is a variation on a dilemma formulated by Plato in his *Meno*. Aristotle's solution to the dilemma of *Meno* is discussed and considered as another plausible strategy for dealing with the paradoxes of inference and analysis.

KEYWORDS: Aristotle – pre-knowledge – the paradox of analysis – the paradox of inference.

0. Introduction

This paper focuses on two remarkable paradoxes related to the subject of rational cognition, namely the paradoxes of inference and analysis. The objective is to investigate the nature of these paradoxes and to somehow resolve

them. The structure of the paper is as follows: first, I briefly introduce both paradoxes and, by comparing them, I come to their common source, pre-existent knowledge. After that, I outline a very interesting conception of this subject as offered by Aristotle in *Analytics*. Then I apply this concept to both paradoxes and, thus, offer it as their common solution. To keep it simple, I try to present all paradoxes in the simplest form possible, *i.e.* in the form of a simple syllogism.

1. Introducing the paradoxes

1.1. Paradox of inference[1]

To keep it simple, this paradox can be presented in the form of the following argument:

Valid inference from true premises is a good tool
for expanding knowledge.
Valid inference does not provide any new knowledge.

Some good tools for expanding knowledge do not
provide any new knowledge.

Commentary on individual parts of the argument:

The first premise ("Valid inference from true premises is a good tool for expanding knowledge"): Many philosophers believe that the tools or means for how we get to know the world around us are our senses and reason. For now, we can leave the manner, competence, and mutual relationship between both means of knowledge aside; the only important matter is that inference definitely belongs among the methods that reason – a tool of knowledge – "works with". Thus, what we assign to rational knowledge as a whole is also relevant for inference. This opinion is also for many people the reason why philosophers (or scientists) should – to some extent – master the discipline which, above all, applies inference, *i.e.* logic. The first premise is, therefore, a compact

[1] The origin of the paradox definitely goes back to Classical Antiquity, its modern form and name were coined by Cohen & Nagel (1934, 173).

expression of this conviction (the word "good" should emphasize that it is a functioning, not a damaged tool).

The second premise ("Valid inference does not provide any new knowledge"): It contains a finding that many philosophers and logicians come to when facing the question of what generally justifies a specific conclusion from given premises. For instance, let us consider the popular syllogism "all men are mortal; Socrates is a man; thus, he is also mortal". We often face the opinion that a conclusion is based on premises because it is in them somehow – implicitly – contained. In our case, Socrates' mortality is given by the fact that as a man he belongs to creatures that the first premise mentions. If the conclusion is contained in the premises before its actual inference, then it seems that the explicit statement of the conclusion cannot provide any new information which would not have already been in the premises, *ergo*, inference does not provide any new knowledge. In this sense, old sceptics – or more recently, J. S. Mill (see Mill 1882, 228) – criticised inference. We can come to the same conclusion by considering the famous Deduction Theorem. It states that every deductively valid argument can be transformed into tautology (in the form of implication where the conjunction of premises forms an antecedent and the conclusion a consequent of implication). In a widely accepted understanding of tautology, it does not provide any new information about the world. If it is therefore possible to equivalently transform deductively valid inferences to tautologies and tautologies do not provide any new knowledge, then also the deductively valid arguments do not provide any new knowledge; *ergo*, inference does not provide any new knowledge.

The conclusion ("Some good tools for expanding knowledge do not provide any new knowledge"): It is correctly inferred from the given premises, specifically it is the syllogistic mood Felapton. It is also seemingly strange; analogically we could, for instance, say that a good tool to hammer nails does not hammer nails. It either is not a good tool for hammering nails and then we should not take it for one or it makes sense and then it can crack down on those nails, despite our opinion – and yet, both simultaneously are impossible.

We, consequently, have premises here which are at least in some philosophical and logical communities quite commonly used, but together they come to an absurd conclusion. A false conclusion, correctly inferred from premises, clearly indicates that (at least) one premise is false; at least one of the given, rather widely accepted opinions is, thus, false.

Given that the second premise seems to contain hardly questionable results of logic, the more common strategy of how to contest the inference paradox is by questioning the truth of the first premise (see the aforementioned ancient sceptics or J. S. Mill). Consequently, we have to revise the opinion regarding the role of inference in human cognition.

The less common strategy is questioning the second premise, *i.e.* questioning the belief that a conclusion does not provide new information. That the conclusion is "somehow" contained in the premises cannot be challenged, therefore they mostly use redefinition or distinction of the terms novelty and/or information.[2]

It is worth remembering that for modern logics the *locus classicus* of the inference paradox is the publication by Cohen and Nagel, *An Introduction to Logic and Scientific Method* from 1934 (see Cohen & Nagel 1934). It defines the inference paradox in the following way:

> If in an inference the conclusion is not contained in the premises, it cannot be valid; and if the conclusion is not different from the premises, it is useless; but the conclusion cannot be contained in the premises and also possess novelty; hence inferences cannot be both valid and useful. (Cohen & Nagel 1934, 173)

Cohen and Nagel thus understand it as a dilemma between validity and usefulness. The concept of usefulness is worth considering – a valid inference is seen useless because it does not provide any new information. Thus, the criterion for usefulness of an inference is that it provides new information. We will see later that Aristotle approached it differently.

1.2. Paradox of analysis

This paradox is most often formulated as a dilemma when analysis is said to be either correct or informative, but never both. Thus, dilemma is surprising because the possibility of a correct and at the same time informative analysis is often considered unproblematic. If converted into a simple argument, we could present it in the following way:

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[2] See, e.g. Duží (2006), or Duží (2010). Needless to say that the redefinition or distinction is not the whole solution, but only a part of the more complex argumentation.

Correct analysis is a good tool for expanding knowledge.
Correct analysis does not provide any new knowledge.

Some good tools for expanding knowledge do not provide any new knowledge.

Commentary on the individual parts of the argument:

The first premise ("Correct analysis is a good tool for expanding knowledge"): Here almost the same applies as in the commentary on the first premise of the inference paradox: analysis, a mental decomposition or a breakdown of a given compound into its constituent components, is understood as one way of how reason – a tool of knowledge – "works". If it holds true that reason is really a good tool for getting to know the world, then the same holds for analysis.

The second premise ("Correct analysis does not provide any new knowledge"): The specific problem here was established by the British philosopher, G. E. Moore (originally in Moore 1903, 7) and it is, let us say, of a semantic nature. It states that if we have, for instance, an analysed concept (an analysandum) and an analysing concept(s) (or analysans), then the basic requirement of a correct analysis is that all that the analysandum contains must be also contained in the analysans and vice versa, thus, analysandum = analysans. If the analysans contained something that was not in the analysandum, it would have been an incorrect analysis. Taking an analogical example from the analysis of physical things, a traditional component of military training was to disassemble a soldier's machine gun down to its components and to reassemble it again. If the soldier conducted the disassembly correctly, then he dismantled the machine gun into and only into the parts that it was composed of – if any additional component appeared, it could not have come from the dismantled machine gun and the disassembly had not been performed correctly (or alternatively, it was a disassembly of the machine gun and something else). If the analysans cannot contain what was not in the analysandum to begin with, then no correct analysis can come up with something new, *ergo*, analysis does not provide any new knowledge.

The conclusion ("Some good tools for expanding knowledge do not provide any new knowledge"): It is completely analogical to the paradox of inference with the modification that the "some" in each paradox targets a different tool for expanding knowledge. Solution strategies are also similar:

The more common strategy is to oppose the first premise, *i.e.,* in some sense revise the view on the role of analysis in human knowledge. This revision – seemingly absurdly – appears in analytical philosophy, that is, a philosophical stream which has analysis in its very title and which focuses its very philosophical work on rigorous analysis. However, the absurdity is only illusory. Analytical philosophers, who reject the first premise of the paradox of analysis, do not wish to claim that the result of their analytical work is new information about the world; they do not wish to compete with sciences. In other words, philosophy – in their understanding – is not a theory but an activity.[3]

Another used strategy is to question the second premise. In this case, it usually means specification of what exactly the equal sign between the analysandum and the analysans relates to: whether to language expressions, meanings, or (non-language) objects depicted by these expressions.

1.3. The two paradoxes compared

The preceding text should anticipate the similarity of both paradoxes. For better illustration, let us present them again, but this time together:

Valid inference from true premises is a good tool for expanding knowledge.
Valid inference does not provide any new knowledge.

Some good tools for expanding knowledge do not provide any new knowledge.

Correct analysis is a good tool for expanding knowledge.
Correct analysis does not provide any new knowledge.

Some good tools for expanding knowledge do not provide any new knowledge.

The similarity of both paradoxes should definitely not be only about the possibility to convert them into almost identical syllogisms. On the contrary, that we can present them in this way is but one indication of their similarity. These similarities are more numerous and deeper:

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[3] See Wittgenstein (2001, 4.112): "Philosophy is not a body of doctrine but an activity".

Above all – both are called paradoxes even though they are clearly different from typical and more famous paradoxes – such as the liar paradox, Russell's paradox, etc. In my view, the main difference lies in the fact that the aforementioned, more typical paradoxes are based on premises or methods, where we cannot easily determine where and if there is any problem at all. They are paradoxes for the very reason that the drawing the absurd consequences from seemingly unproblematic premises is striking; it is not clear what is wrong. Our paradoxes often constitute some logical-noetic intuitions shared by a community but always rejected by many other experts. These statements are not unproblematic and their rejection does not seem that shocking or fatal. They are paradoxes almost literally – the term "paradox" refers to a situation, when various *doxai*, *i.e.*, opinions or intuitions, go "against each other". In our case it is the contradiction of various intuitions related to the role of rational knowledge, thus, being incompatible, together forming an absurd result. Therefore, it concerns the divisions within the community of logicians and philosophers.

Furthermore – both paradoxes' first premise is always a general statement on the role of the relevant component of reason for knowledge, while the second premises capture a finding of a more logical or semantic nature.

Thirdly – the second premise always captures a logical-semantic piece of knowledge, *i.e.*, from a sphere that enjoys a relatively high level of authority. This fact gives rise to the attempts to solve the paradoxes by criticizing the first, more of a philosophical (specifically noetic) premise.

Fourthly – paradoxes form relatively simple arguments, where it is difficult to question the fact that the conclusions really follow from these premises, thus, the attempts to question the second premise most commonly use the method of concept distinction, specifically concepts of novelty, information or equality.

Fifthly – the most significant similarity is related to a problem which is difficult and is the proper subject matter of both paradoxes. I believe it stands behind the reason why so many contradictory beliefs can arise about the same thing – it is the question of implicit knowledge or pre-existent knowledge. For both paradoxes, this problem is hidden in the second premise. In the case of the paradox of inference as stated above, the conclusion follows from the premises for the very reason that it is somehow – implicitly – contained in them. Yet, what does this really mean? If the conclusion is "in some way" in the premises, then this could mean that every validly inferred argument incorporates a mistake of the so-called reasoning in a circle, which is a situation when

the conclusion is one of the premises. Circular reasoning is a deductively valid argument, but is considered as faulty because it does not contain any new information compared to the premises, *i.e.*, is trivial. If the fact that the conclusion of every deductively valid argument is somehow contained in the premises means that the conclusion appears in the premises then every deductively valid argument is at the same time circular reasoning, trivial and uninformative. On the other hand, we have rich experience, especially with more complex arguments, arguments with more and/or complicated premises where the conclusion is really surprising and we are inclined to believe that the conclusion is new information for us. Thus, it seems that the situation when the conclusion is implicitly contained in the premises is not simply a situation where the conclusion is one of the premises as would be the case of circular reasoning. Information contained in the conclusion is somehow already present for us in the premises, but at the same time we know it in a somewhat different way than we know the premises.

In the case of the paradox of analysis, we can understand analysis generally as decomposition, as breakdown of a whole into its components. When decomposing a whole, we have to have this whole available, it has to be somehow given to us – as a whole with all its components. Analysis can be correct for the very reason that if we are decomposing a given whole and not something else, then what we get are parts of this whole and not something else. This should, however, mean that a correct analysis cannot provide anything new because the parts obtained by analysis had already been somewhat available when we had the whole thing in front of us prior to commencing the analysis. It is as with the soldier who disassembles his machine gun – before he starts the dismantling, he has a machine gun in front of him, *i.e.*, the whole with all its components. When he starts to disassemble the machine gun, then everything he gets, for instance lock frame or striker, he had already somehow at his disposal at the moment when he started the dismantling. On the other hand, analysis of more complex concepts or statements can really strike us, it might surprise us that some parts of the analysans belong to this very analysandum.

The situation is, therefore, similar to that of the paradox of inference – *on one hand the "result" (conclusion or analysans) is already somehow present in the given (premises or analysandum) and this very presence is the guarantee of correctness (of inference or analysis); on the other hand, the presence is such that its (re)emergence "in the result" can surprise us and is thus present only indirectly*. We somehow know the conclusion and analysans in advance,

this *pre-existent knowledge* is a special type of knowledge which deserves special attention. An insight into the nature of this pre-existent knowledge can simultaneously provide a way how to better understand the challenges of both paradoxes or even more provide a solution.

2. Aristotle's theory of pre-existent knowledge

In the next part, I would like to offer a very interesting – and, as far as I can see, also plausible – theory of pre-existing knowledge that Aristotle presented in some of his logical treatises. I would like to defend the plausibility of Aristotle's concept by its very ability to solve the paradoxes of inference and analysis which, based on what was said above, I consider derived from the problem of pre-existing knowledge.

2.1. Plato

Aristotle "inherited" this problem like many others from Plato. Given that Plato's answer to this problem is his famous doctrine of ideas, we can claim that it could not have been a marginal issue for him. Thus, it is apt to further investigate how he handled it. It allows us to find out what, specifically, Aristotle responded to and in what form he took it over from Plato.

We can find a very concise introduction into the problem in Plato's dialogue *Meno*. Socrates discusses with Meno the nature of virtue and demonstrates to him the unsustainability of the definition of virtue as presented by Meno. Meno is at a loss as to how, and if at all, to continue. Socrates encourages him and maintains that we should continue to pursue the definition of virtue. Yet, Meno objects:

> Why, on what lines will you look, Socrates, for a thing of whose nature you know nothing at all? Pray, what sort of thing, amongst those that you know not, will you treat us to as the object of your search? Or even supposing, at the best, that you hit upon it, how will you know it is the thing you did not know?
> I understand the point you would make, Meno. Do you see what a captious argument you are introducing—that, forsooth, a man cannot inquire either about what he knows or about what he does not know? For he cannot inquire about what he knows, because he knows it, and in that case is in no

> need of inquiry; nor again can lie inquire about what he does not know, since he does not know about what he is to inquire. (Plato, *Meno* 80d-e)

All in all – it states the following dilemma: if we want to know something, then we have two options – we know or we don't know the thing which we are discovering. If we know the thing we are discovering, then it makes no sense to commence with the discovery because we know it already. If we don't know the thing we are discovering then even if we would come across it during our discovery, we would not know that it was the thing we sought. Thus, even in this case, it makes no sense to commence with discovery.

The answer to this "captious argument" is in this dialogue the doctrine of ideas – Plato reinterprets cognition so that putative knowledge (compromised by Meno's dilemma) is in fact re-cognition because in its quest the soul starts to remember the idea of the object which it is looking for.

2.2. Aristotle's solution

It is well known that Aristotle rejected Plato's doctrine of ideas which meant that he, besides other things, had to readdress the Meno paradox. He explicitly handled this problem mostly in chapter 21 of *Prior Analytics*, *Book II*, and in the first chapters of *Posterior Analytics*.[4] Like Plato, Aristotle reinterprets the common term cognition whose limits were rightly accentuated by Meno's paradox. He, however, did not claim that cognition is in reality re-cognition and distinguished between several meanings of the term "knowledge". The starting point was for him one of the basic dichotomies of his philosophy, potentiality and actuality. His *Posterior Analytics* start with the sentence: "All instruction given or received by way of argument proceeds from pre-existent knowledge". Thus, Aristotle – like his teacher – accepted some knowledge preceding one's own cognition.

Instead of identifying this pre-existent knowledge with the recollection of ideas, he understood this *pre-existent cognition as potential knowledge*, or knowledge in possibility, which only in the process of cognizance becomes true actual knowledge. In *Prior Analytics*, Aristotle even distinguished between three meanings of knowledge: knowledge of what is general (further called universal knowledge), what is particular (particular knowledge) and

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[4] The presented interpretation of Aristotle's solution was taken mostly from Mráz (2000) and Barnes (1978).

knowledge of what is actual (cf. An.Prior. II, 67a33-b5). For a better illustration and deeper understanding of this concept, we use Aristotle's own examples. Aristotle considered (in the chapter mentioned) these arguments:

Every triangle has two right angles.[5]
This is a triangle.
———————————————
This has two right angles.

Every mule is infertile.
This is a mule.
———————————————
This is infertile.

Both have the same logical form:

$$\frac{\forall(x)(\mathrm{P}(x) \rightarrow \mathrm{Q}(x)) \quad \mathrm{P}(a)}{\mathrm{Q}(a)}$$

In both, every judgment represents one type of knowledge. The first one is universal knowledge, the second is particular knowledge and the conclusion is actual knowledge. We will step-by-step discuss these individual types of knowledge:

Universal knowledge. Question is how we obtain this kind of knowledge. In Aristotle's case, we can talk about, for example, proof or incomplete induction. These topics are very delicate from the point of modern epistemology but we do not need to further follow them for our purposes.

What matters is that in terms of a given argument we do not know the "general" premise in the same way that we know the conclusion, *i.e.*, in terms of the given argument it is not justified knowledge (or in other words – it is not knowledge justified by the given argument). Simultaneously, we do not have at our disposal all the subjects of universal knowledge. Aristotle demonstrated it in chapter 1 of *Posterior Analytics* in the example of a sophisticated "trick" which had probably been known in his times. The point was to present the

[5] Aristotle wanted to say that all interior angles of a triangle add up to two right ones.

opponents a sentence "every couple is even" and ask whether they knew it. If they said no, then they would be easily shown that if they understood the term couple well, then they would have had to admit that a couple was even. If they said yes, then they would be presented with a couple that they did not know it was a couple and therefore they also did not know that this couple was even. As Aristotle stated, one suggested solution was to say that if someone claimed that every couple is even, then they actually only said that every couple they know is even. Aristotle rejected this *ad hoc* constriction and insisted that the statement is really relevant to all couples, not only those that the speaker know. Here comes the point – related to the couples that the speaker did not know, the statement "every couple is even" is potential knowledge. That means that he did not actually know it about these couples, but if he realized it was a couple then he would have known it was even. Thus, potential knowledge becomes actual knowledge. In relation to these subjects it is potential knowledge, pre-existent knowledge! *Pre-existent knowledge is potential knowledge*, which is in some way at our disposal, but we always need something else to turn it into actual knowledge.

Particular knowledge. Unlike the aforementioned, this knowledge regards only one thing. What we know about it is that it falls under some universal, or that we recognised this individual as falling under some general determination. Using modern logics, it is a finding that an individual *a* lies in the domain of predicate *P*, while according to the first premise, the domain of *P* is a subset of domain *Q*.

Actual knowledge. It is knowledge which unites the preceding universal and particular knowledge. It is based on *Aristotle's notion that actual knowledge is knowledge of causes or reasons,* thus, actual knowledge is (ideally) the conclusion of an argument. The given premises are the reasons why we actually know what the conclusion states. Actual knowledge is, therefore, justified knowledge and for Aristotle also knowledge of what is necessary.

In order to know what the conclusion says we must know the premises, *i.e.*, think them simultaneously and in mutual correlation. If not, then the conclusion would simply be an accidental statement. Aristotle had this in mind when mentioning both arguments in *Prior Analytics*. If we, for instance, knew that every triangle had two right angles, it would not justify the statement that this had two right angles – we would lack the knowledge that it was a triangle. Similarly, if we only knew it was a triangle, then we could not justify that this had two right angles. Even if we knew that every triangle had two right angles

and simultaneously that this was a triangle, it would not mean that we knew that this had two right angles. We have to actually think both findings simultaneously and in mutual correlation. Only this correlation establishes a justifying relationship between the premises and the conclusion.

2.3. Summary

It is a good idea to sum up here what has been said so far. When investigating the paradoxes of inference and analysis, we discovered many common points and most importantly we uncover the problem of pre-existent knowledge as a common source of both paradoxes. I outlined as a plausible concept of pre-existent knowledge Aristotle's theory, which rests on distinguishing three types of knowledge: universal, that is, only potential knowledge, particular knowledge and actual knowledge, which is justified knowledge connecting both previous types, thus, (ideally) the conclusion of the argument. Pre-existent, universal knowledge is therefore (a kind of) potential knowledge. It is neither unknowing nor actual knowledge (thus, none of the Meno's paradox possibilities), but something in the middle: knowledge in possibility.

We should also mention in connection with Meno's paradox that it emerges in relation to universal knowledge. If, for instance, the argument's premises represent particular knowledge (and conclusion actual knowledge), then this dilemma will not arise.

I will now try to apply this Aristotle's concept to both paradoxes.

3. Paradoxes revisited

3.1. Paradox of inference revisited

To remind the reader, the paradox of inference was presented in the form of this argument:

Valid inference from true premises is a good tool
for expanding *knowledge*.
Valid inference does not provide any new *knowledge*.

Some good tools for expanding knowledge do not
provide any new *knowledge*.

The argument repeats the concept of knowledge several times. After listing Aristotle's classification of types of knowledge, it is worth mentioning which types of knowledge refer to which place.

The second premise ("Valid inference does not provide any new knowledge"): As mentioned in section 1.1., inference does not provide any new knowledge because the conclusion of the sound argument is somehow (implicitly) contained in the premises. It is crucial to grasp its implicit presence in the premises. This is quite difficult to do and so we will use examples. Possibly the best way to demonstrate how the conclusion is contained in the premises is to use arguments of modus ponens form. Let's take argument (a): If this animal is a mule, then this animal is infertile. This animal is a mule. Thus, this animal is infertile. Conclusion of (a) – this animal is infertile – is even to a naked eye contained in the premises. It is not included, though, as an independently standing premise making such a statement. That would make (a) reasoning in a circle and I surely do not wish to claim that every valid argument is reasoning in a circle. Conclusion of (a) is contained in the premises as a consequent of implication (premise one). That means that conclusion of (a) is stated only conditionally, *i.e*., in order to say that, we would have to know whether the respective condition was met. The first premise does not say that though and we, therefore, cannot find out from the first premise only whether the animal is infertile. This judgment is somehow available to us, but is not stated; we do not know whether it is true. In Aristotle's terms, it is not actual knowledge.

Somewhat less visible is the conclusion of an argument contained in the premises in case of the argument of modus tollens form. Let's take argument (b): If this animal is a mule then this animal is infertile. This animal is not infertile. Thus, this animal is not a mule. Conclusion of (b) – this animal is not a mule – is not already contained in the premises in such an obvious way as was the case of conclusion of (a). This judgement is in the premise as a condition, thus, is not itself stated, it is in a similar "situation", as in case (a), the same judgement is in the conclusion even negated. Not only we have this judgement somehow available to us in the premises but is not stated; we don't know if it is true, but it is also in the conclusion negated as untrue.

Even less obvious is the presence of the conclusion in the premises of the famous syllogism (c): All men are mortal. Socrates is a man. Therefore, Socrates is mortal. Conclusion of (c) – Socrates is mortal – is to the naked eye not at all contained in the premises! Only its "parts" are included, in the second

premise subject, in the first predicate, the matter of its affirmation or negation, thus, does not arise at all. What the first premise of (c) says is that the predicate of mortality belongs to all men, but it neither mentions a specific man, nor talks about any specific man, thus, neither mentions Socrates. This premise states that the predicate of mortality is common to men, thus, any man. If someone agrees with this premise, then he/she does not claim that the predicate of mortality belongs only to men he/she knows. It is the same situation as "every couple is even" mentioned above (cf. section 2.2.). If someone agrees with the statement that all men are mortal, then he/she agrees that this judgment relates also to entities he/she does not know, that is, to entities he/she does not know that they are also people. If then he/she does not know that, for instance, someone called Socrates is a man then regarding this Socrates the knowledge that he is mortal is only knowledge in possibility. It is not complete unknowing because the knowledge of first premise *allows* us to gain knowledge that Socrates is also mortal. If the first premise was replaced with for instance a judgment "every mule is infertile", then the first premise would not allow us to gain knowledge that Socrates is also a man.

All in all – the implicit presence of a conclusion in a premise means that this premise offers potential knowledge of (future) conclusion, thus, its preexistent knowledge. That a given judgment appears in the conclusion of a valid argument is not extension of potential knowledge. This potential knowledge was available already in one of the premises. We can rephrase the second premise of paradox of inference as: Inference does not provide any new potential knowledge.

The first premise ("Valid inference from true premises is a good tool for expanding knowledge"): If we recognize that inference does not provide any new potential knowledge, then how to avoid the conclusion that inference is epistemically useless and does not provide any new knowledge? We hinted the answer in the preceding analysis of the first premise of inference paradox – we know the argument's conclusion differently than as shown/hidden in the premises.

To be more rigorous: to know that the conclusion is contained in the premises actually means that we know how to infer it from the premises. Such (explicit) knowledge, thus, means that we thought of both premises at the same time and in mutual correlation which Aristotle stated as a condition for actual, thus justified, knowledge. We saw above that the potential knowledge of the conclusion in the premises is something that requires addition – with modus

ponens this is the knowledge of the antecedent's truth, with modus tollens the knowledge of the consequent's falsity, with our syllogism the knowledge that someone called Socrates is also a man. After this addition, knowing the conclusion is knowledge of different kind because the interconnected knowledge contained in the premises (thought simultaneously and in mutual correlation) provides a reason for affirming the conclusion. As late as now the conclusion becomes a real conclusion and the justifying knowledge become premises. Strictly speaking, Aristotle's actual knowledge is therefore not new knowledge because potential knowledge must always precede this knowledge, but it is knowledge of different kind. If we revisit Cohen and Nagel's formulation of the paradox of inference, valid argument is not useful according to them because it does not provide any new knowledge. In line with Aristotle's reasoning, we can reply that we know the conclusion of the argument differently than how we know the premises, *i.e.*, in a justified manner. The conclusion does not provide any completely new knowledge but provides this knowledge in a different way. The utility of the valid argument, thus, rests on how it is given to us – it is different to have some knowledge and to have some knowledge together with its justification. Expansion of actual (not potential) knowledge, hence, means expanding knowledge, which is given in a justified manner.[6]

We can therefore reword the first premise of the paradox of inference as: inference provides new actual knowledge.

The original paradox of inference will, accordingly modified, look like this:

Valid inference from true premises is a good tool
for expanding actual knowledge.
Valid inference does not provide any new potential knowledge.

Some good tools for expanding actual knowledge
do not provide any new potential knowledge.

The difference from the original wording is fundamental and it says that the premises that are true lead to a correctly inferred conclusion that is also true. Thus, paradox is eliminated.

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[6] In modern epistemological logics, this distinction corresponds with the difference between explicit and implicit knowledge (I would like to thank an anonymous reviewer of this paper for mentioning this).

3.2. Paradox of analysis revisited

As a reminder, the paradox of analysis was presented in the following argument:

Correct analysis is a good tool for expanding *knowledge*.
Correct analysis does not provide any new *knowledge*.

Some good tools for expanding knowledge do not provide any new *knowledge*.

Similarly to the paradox of inference, the concept of knowledge also appears here several times. After listing Aristotle's categorization of knowledge types, it will be interesting to see which types of knowledge belong where.

The second premise ("Analysis does not provide any new knowledge"): As mentioned in section 1.2., equality between the analysandum and the analysans means that correct analysis cannot provide anything new. Correctness of the analysis is given by the fact that analysans is already in some way (implicitly) contained in the analysandum. Thus, it is crucial to somehow more thoroughly grasp its implicit presence.

Let us again begin with examples – we mentioned above the analogy of disassembling a machine gun. In the case of a machine gun, I have in front of me a specific whole which I can take apart into pieces (and let us assume that I have never done it before and did not receive any theoretical instruction about the components of a machine gun). When I had the machine gun in front of me, I somehow also had all its components in front of me. If I dismantled it correctly and dismantled only this machine gun, I really have in front of me all its parts. I have in front of me parts after the dismantling of the machine gun which I had somehow in front of me when I had the machine gun in front of me. What is the difference here, why do I use in the first case the vague expression "somehow"? Simply because in the first case, I could see only some parts of the machine gun, for instance, trigger, but others such as the striker were hidden from my view. Even before dismantling I considered the machine gun as a whole and due to its function expected it to contain other parts than those I could see when having a machine gun in front of me as a whole. Thus, I knew that it contained also parts which I could not see before dismantling it. Furthermore, I knew that those parts invisible to me would be in some relation to those which I could see and to each other. However, only executing the

dismantlement clarified what the hidden parts were and how they were located in relation to each other, for example when I found a striker and learnt about its relation to the trigger. Thus, I knew already before the dismantlement that there were parts in the machine gun which somehow caused the bullet to leave the gun barrel, but I knew neither what these parts were nor what their relationship to each other parts were. The knowledge of these parts before the dismantlement was, thus, only potential knowledge, pre-existent knowledge, actualised by executing the dismantlement.

If we leave the analogy with a physical analysis, we can use the examples of sentences and their analysis. Let us take the sentence (a) "Every man is mortal"; according to classical modern logics, we would find out after analysis that (a) could be rephrased as: "For every individual, if it is a man, then it is mortal." It is important here that between "man" and "mortal" is according to this analysis a relationship of implication which could be expressed in a phrase such as "if …, then …". Before the analysis, I had in front of me a whole (a) composed of certain parts. These parts also appeared in the analysans of (a). What the analysandum did not explicitly contain was a phrase "if…, then…", in other words, it was not obvious to a naked eye that according to this sentence between man and mortality was a relationship of implication. Even before the analysis, after learning the analysandum, I knew that there was some relationship between man and mortality in the analysandum though expressed by a somewhat ambiguous "is". Only the correct analysis of the statement found out that this relationship is one of sufficient condition when being a man is a sufficient condition for being mortal. Who learns about the analysandum can, thanks to this acquaintance, know that the relation in question is a relationship of sufficient condition. Only after the analysis process, however, has explicit knowledge of structure (a), thus, has actual knowledge. Knowing analysandum without knowing the analysis is, hence, potential knowledge of the sentence's structure. It is not complete unknowing, let us say it is a rather necessary (but not sufficient) condition for the explicit knowledge of implication structure (a).

Even less evident is the analysans' presence in the analysandum in Russell's famous analysis of the sentence (b) "The present king of France is bald". According to Russell, (b) actually contains three sentences connected by conjunction. We could simplify it a lot and put the analysans (b) in the following way: "There is an individual who is presently the king of France and there is only one such individual and this individual is bald". If Russell's analysis (b)

is correct, then (b) implicitly contains a conjunction of these three sentences. This knowledge is only implicit, or potential, because (b) contains neither connective "and" which is a sign of a conjunction nor other components of the analysans. Yet, it is not simple unknowing as indicated also by the fact that in case of non-existence of the king of France or in the matter of proper negation (b), the person not having the knowledge of correct analysis (b) can be at a loss. The analysandum, without knowing its analysis, does not in these cases offer clear answers because some parts of the whole (b) and their mutual relations (conjunction here) are hidden in the analysandum. It is hidden analogically to how the striker was hidden in case of the machine gun. Modern logic talks about the grammatical form of the sentence not corresponding with its logical form.

In both examples provided, the implicit knowledge of the whole is a different type of knowledge than explicit knowledge of its parts and their mutual relations. That the analysandum and the analysans contain "the same" (and analysis then does not provide any new knowledge) needs to be corrected. The second premise of the paradox of analysis could be rephrased as: Analysis does not provide any new potential knowledge.

The first premise ("Analysis is a good tool for expanding knowledge"): If we speak about analysis as expanding knowledge, it is clear that we do not mean potential knowledge but justified knowledge, or actual knowledge. If we say, for example, that part of a concept (analysandum) is a part *X*, then it is so to say a snapshot. If we, however, say it after executing an analysis then it is (by a process of correct analysis) justified knowledge. The statement that part of a given concept is component X will be the same in both cases (thus, it is not completely new knowledge), yet in the second case it is justified knowledge. Most importantly though, we emphasised above that many parts of the analysed wholes cannot be simply detected from these wholes, even more so in case of the parts' mutual relations. The first premise of the paradox of analysis can be therefore rephrased as: Analysis provides new actual knowledge.

The original paradox of analysis will after modification look like this:

Analysis is a good tool for expanding actual knowledge.
Analysis does not provide any new potential knowledge.

Some good tools for expanding actual knowledge
do not provide any new potential knowledge.

The difference from the original wording is substantial. It states that a correct conclusion is inferred from premises which are also correct. The paradox is eliminated.

4. Conclusion

The paradox of inference and paradox of analysis have both been understood in our research as grounded in the problematics of pre-existent knowledge. The clarified concept of pre-existent knowledge as presented by Aristotle offers the opportunity to understand both paradoxes as based on a vacillating concept of knowledge which leads some authors to opposite opinions. Further specifications of this concept leads in contrast to a discovery that these opposing opinions refer to different types of knowledge and as such do not actually need to be contradictory and do not lead to a paradox.[7]

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