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Mathematical Models as Abstractions

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ABSTRACT: The paper concerns a contemporary problem emerging in philosophy of science about the explanatory status of mathematical models as abstractions. The starting point lies in the analysis of Morrison's discrimination of models as idealizations and models as abstractions. There abstraction has a special status because its non-realistic nature (e.g. an infinite number of particles, an infinite structure of fractal etc.) is the very reason for its explanatory success and usefulness. The paper presents two new examples of mathematical models as abstractions – the fractal invariant of phase space transformations in the dynamic systems theory and infinite sets in the formal grammar and automata theory. The author is convinced about the indispensability of mathematical models as abstraction, but somehow disagrees with the interpretation of its explanatory power.

KEYWORDS: abstraction – dynamic systems theory – explanation – formal grammar – idealization – mathematical model – Morrison – philosophy of science.

1. Introduction

I believe that in the current debate on the nature of scientific models the traditional question (typical for the semantic conception of scientific theories) of the relationship between abstract models and theories, has been

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somewhat neglected. The current mainstream debate on the nature of scientific theories is commonly referred to as a pragmatic view of theories. This debate was launched primarily by Nancy Cartwright (1983; 1999) and Ronald Giere (1999; 2006), and can be summarized as an approach resigning to the description of scientific theory as an abstract structure with clearly defined relations between the individual components within this structure. Theory is simply conceived as a cluster of models that are appropriate to represent certain elements of the phenomena under investigation. Currently even the very idea of scientific theory is neglected in favor of the idea of scientific modelling (see e.g. Gelfert 2016, Zach 2017).

Scientific models, e.g. causal, non-causal (and plethora of their types), in this context are fruitfully investigated in terms of building the typology of models and in terms of important contributions to topics of scientific explanation and prediction (see e.g. Weisberg 2013). Yet I think this omits an important question central in the traditional philosophy of science. This question cannot be ignored, and is eventually testified by some texts of the proponents of the pragmatic conception of theories themselves, especially by Ronald Giere. In "Scientific Perspectivism" he modified his pragmatic conception of theories when, in addition to the introduction of data models, he conceded within the abstract models a definite place for principles (see Giere 2006, 61-62). However, Giere neglected the question of the nature of the nexus between principles and models.

The pragmatic view of theories works with models primarily as idealizations that are appropriate to represent a particular situation (for the researcher/scientist, see Giere 2006, 60, 62-63), given that they are similar to the data models investigated as "operationalized events /entities". The question of defining similarity (see Giere 2006, 63-67) as a sufficiently precise[2] concept will be shelved and we will focus on the view of mathematical models as abstractions.[3]

The aim of the study is to develop the concept of mathematical model as abstraction offered by Margaret Morrison. Her approach is inspirational because it overcomes the constraints imposed by the current concept of simplifying assumptions. This allows us to avoid the pitfalls of fictionalism

2 Peter Smith accuses Ronald Giere of vagueness, see Smith (1998a, 253-277).

3 We are aware about the debate concerning simplifying assumptions of scientific models, which are defined as abstraction and idealization, see Godfrey-Smith (2009).

and formalism (together we can call them mathematical utilitarianism), but also realism (or mathematical Platonism) in approaching mathematical models in the natural sciences (especially in physics).

However, although Morrison points at the peculiar position of mathematical models as abstractions, she does so only with the help of a relatively limited set of examples (linked to the renormalization group). In addition, she faces the problem of combining of unrealistic properties of explanatory models with an explanatory theory. This second problem is more serious because it can lead to a leap (rejected by Morrison) to the explanatory power of mathematics itself in relation to a natural science.

The first problem will be removed by presenting other two examples in which abstraction plays a crucial role. The first example is from the dynamic systems theory, the other example comes from linguistics, particularly from the field of formal grammars. The definition of the concept of abstraction by Morrison and the introduction of two new examples will be elaborated in the second and third sections of the study.

The fourth section will focus on the second issue of Morrison's approach. We will outline how to prevent the mentioned danger, through a close alignment of mathematical models as abstractions with their theoretical principles (which is also present in two new examples). The rehabilitation of the concept of the theoretical principle leads to the fulfillment of the explanatory potency of a scientific model (in our case: of abstraction).

2. Mathematical models as idealizations and abstractions

Morrison inclines towards pragmatic and pluralistic view of theories based on scientific models; she says that models act as autonomous mediators between theory and applications, or between theory and the world (see Morrison 2015, 20). However, in contrast to pragmatic-oriented variants of classical model-based views of theories (MOT) she fundamentally modifies the meaning of specifically mathematical models in this mediation by distinguishing their role according to whether they are abstractions or idealizations.[4]

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[4] We have to notice that the way of using the term abstraction and idealization is slightly different from usage in context of simplifying assumptions.

Morrison states:

> (…) abstraction is a process whereby we describe phenomena in ways that cannot possibly be realized in the physical world (…); the mathematics associated with the description is necessary for modelling the system in a specific way. Idealization on the other hand typically involves a process of approximation whereby the system can become less idealized by adding correction factors (…) idealization is used primarily to ease calculation. (Morrison 2015, 20)

The last sentence reminds us of the classic MOT, which is characteristic of Ronald Giere where models are actually viewed as useful tools used to represent aspects of the world:

> What is special about models is that they are designed so that elements of the model can be identified with features of the world. This is what makes it possible to use models to represent aspects of the world. (Giere 2004, 747)

Morrison adds:

> In their original state both abstraction and idealization make reference to phenomena that are not physically real; however, because the latter leaves room for corrections via approximations, it can bear a closer relation to a concrete physical entity. (Morrison 2015, 20-21)

For this reason, models like idealization are favoured by most MOT supporters. Morrison, however, shows us that this view of the model and of its role in scientific theories are both extremely simplified. Morrison focuses on those cases of applying mathematical abstractions in models where these abstractions are not accessible to approximation techniques (see Morrison 2015, 21). Because, according to Morrison, these abstractions are necessary to depict and understand the behavior of physical systems, of which she says: "(…) the inability of standard accounts to capture the way mathematical abstraction functions in explanations" (Morrison 2015, 21).

Morrison comprehensively investigates the role of such abstractions, both in terms of their ability to provide general features of physical systems (see Morrison 2015, 25-26), and, for her more importantly, in terms of their ability to provide: "(…) detailed knowledge required to answer causal questions" (Morrison 2015, 26).

The chief example chosen by Morrison is the dynamics of phase transitions:

> The occurrence of phase transitions requires a mathematical technique known as taking the "thermodynamic limit" N → ∞ (…), we need to assume that a system contains an infinite number of particles in order to explain, understand, and make predictions about behaviour of real, finite system. (Morrison 2015, 27)

Morrison points out that this is not a kind of simplistic calculation but:

> (…) the assumption that system is infinite is necessary for the symmetry breaking associated with phase transitions to occur. (…) we have a description of a physically unrealisable situation (an infinite system) that is required to explain a physically realisable phenomenon (the occurrence of phase transitions in finite systems). (Morrison 2015, 28)

I believe that Morrison's fundamental insight into the exclusivity and indispensability of mathematical abstractions as a means of theoretical representation (see Morrison 2015, 29) is marred through excessive affinity of most of the cited examples to "emergent phenomena" (see Batterman et al. 2013). These are also closely related to phase transitions in connection with dynamic systems theory (hereafter DST, see section below). Moreover, when Morrison talks about the use of mathematical abstractions in biology, they occur in areas that are linked to DST (population dynamics). Taking into account other major Morrison texts, this becomes even more clear, because all the examples mentioned fall within the scope of scientific unification through universality (see Morrison 2013, 381-415).[5]

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[5] Morrison even distinguishes three variants of unification of theories: through reduction, synthesis and on the base of universality.

The specificity of mathematical abstractions that even provide information[6] on the physical (or biological) system under investigation (see Morrison 2015, 55) is demonstrated through the example of a renormalization group (RG), which is used for the mathematical modelling of the dynamic system at critical points in phase transitions (see Morrison 2015, 57-67). These descriptions lead Morrison to DST and to the concept of universality:

> Diverse systems (…) with the same critical exponents exhibit the same critical behaviour as they approach critical point. In the sense they can be shown via RG to share the same dynamic behaviour and hence belong to the same universality class. (Morrison 2015, 70-71)[7]

Morrison talks about the ontological independence of the macro level of description at the micro level of description (see Morrison 2015, 74) and conveys the need to formulate a new concept of scientific explanation:

> Instead of deriving exact single solutions for a particular model, the emphasis is on the geometrical and topological structure of ensembles of solutions. Further explication of these aspects of RG methods allows us to appreciate the generic structural approach to explanation that RG provides. (Morrison 2015, 76)[8]

As evidenced by the citations, the whole discussion about the abstractions at Morrison concentrates on DST. In the first part of the next section,

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[6] This is a rather vague part of Morrison's argumentation, where on the one hand it cannot be said that mathematics can provide an explanation of physical facts, but on the other hand it cannot be claimed that information about the physical system is included entirely in the physical hypothesis (and in specific conditions). Thus, mathematics acquires a specific status not only as a means for explaining but also as a co-constituent of information on the system under examination (see Morrison 2015, 55).

[7] Morrison also recalls the importance of power laws to describe regulatory parameters. She recalls a number of variants of these laws across disciplines (see Morrison 2015, 70). We should recall that they are also important in the context of quantitative linguistics (see Köhler et al. 2005).

[8] Significant similarity to Kellert's concept of qualitative prediction and description of geometric mechanisms (see Kellert 1993, 97-105).

abstraction defense will be used directly in the DST context on the level of application of fractal geometry. This shows the issues of phase transitions, critical points and the use of RG in another perspective, following the discussion by Stephen Kellert and Peter Smith.

To demonstrate that the importance of mathematical abstractions for understanding (or even for explanation) within scientific theories is not only tied to DST, we also provide a second part of the third section exploring the abstractions beyond DST and physics. We will focus on the importance of the mathematical model of an infinite set for automata theory and formal grammar.

3. In support of abstractions

3.1. Fractal geometry in dynamic systems theory

Dynamic Systems Theory (DST) is one of the central scientific concepts on which a large part of today's scientific applications, and new theoretical approaches rests. The debates of philosophers of science on the DST culminated in the 1990's and was predominantly formulated by Stephen Kellert (1993) and Peter Smith (1998). This theory, especially under the popularised name chaos theory, was in the focus of the philosophers of science for reasons connected with a pronounced relativisation of methodological criteria in the natural sciences. Foremost was the discussion about the revision of some important philosophical-scientific concepts – especially scientific law[9] in the context of the views of scientific theories and predictions within scientific explanations.[10]

The degree of change effected by DST, judging representatively on the basis of Kellert's and Smith's texts, is not too extensive and is well documented. Unfortunately, Smith's interesting idea of the importance of fractal geometry for the explanations of dynamic behaviour of the system, which is in a chaotic mode, has been largely unnoticed. We cannot reasonably

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9 Here we draw attention to Kellert's inspiration by Giere's studies of the 1980s.

10 Today, the main debate is concerned with the issue of phase transitions and the associated universality of the description of phase transitions across various scientific ontologies. This also often involves the concept of emergence (see e.g. Batterman ed. 2013).

expound fractal geometry and its application to DST (see Peitgen et al. 2004). However, two aspects of this mathematical entity are essential for our purpose; the first is the infinity of the fractal structure and differentiating fractals from prefractals.

In the DST the concept of infinity was crucial. Its importance is appropriately summarized in the redefinition of Laplace's demon postulation. In order to allow unlimited predictions of the evolution of the dynamic system over time, in some cases (for certain control parameters) we need to know accurately all initial conditions of the dynamic system.[11] In short, Laplace's proverbial demon must indeed possess an infinite memory and omniscience.

This interpretation of the predictive constraint in DST is reflected in Kellert's concept of the transcendental impossibility of certain types of predictions (see Kellert 1993, 32-42). We refer to it here because we think it contrasts with the correct use of the mathematical model as an abstraction in the case of Smith. In the case of Kellert, an abstraction of infinite precision is used because the theory can demonstrate that for an arbitrary little inaccuracy of knowledge of the initial conditions, we always find (in the case of chaotic dynamics) the situation in which the error rate reaches the magnitude of the measured quantity. In other words we lose the ability (quantitative) to predict development of the system (sensitive dependence on initial conditions).

I believe that the abstraction of Laplace's demon with the infinite memory is inadequate, because the need to know all the details of a dynamic system is dispensable. From the empirical point of view, it makes no sense to think that the degree of inaccuracy is infinitely small, but it will be reflected in the final instance. The use of the infinity model is therefore in this case only idealization.

Similarly, when we use fractal geometry in many cases, it is enough to build on the knowledge of the most suitable prefractal without needing to work with the infinitely fine structure of the fractal. Analogous to Morrison's examples, the mathematical object of the fractal is used as an ideal object for only a certain aspect of creating a hypothesis (in relation to representation of the data model), to a certain level of accuracy (the number of iterations performed). Analogously, for example, because we know that

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11 Prigogine discusses this in "Order out of Chaos" (1984).

the sea border of Norway is not infinitely long, we do not need to revert to the molecular or even atomic level to describe the structure of its coast.

It seems that prefractals are therefore a good example of mathematical models as idealizations, as Morrison discusses. In this case, the mathematical object is not present in the theory or application of the theory as a whole, but only its appropriate scheme. Smith, however, also focuses on mathematical DST models that clearly correspond to how Morrison characterizes mathematical abstraction. Smith expresses the core of the problem in a simple argument:

> To summarize: we initially noted that
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> (F) The chaotic behaviour in models like Lorenz's depends on trajectories getting pulled ever closer to a strange attractor with a fractal geometry.
>
> It has now been argued that
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> (G) The evolving physical processes that chaotic dynamic models like Lorenz's are characteristically intended to represent cannot themselves exhibit true infinite intricacy.
>
> (F) and (G) together imply the conclusion that, at least in the typical case, the very thing that makes a dynamic model a chaotic one (the unlimited intricacy in the behaviour of possible trajectories) cannot genuinely correspond to something in the time evolutions of the modelled physical processes – since they cannot exhibit sufficiently intricate patterns at the coarse-grained macroscopic level. (Smith 1998, 41)

Still, according to Smith, we find cases (see Smith 1998, 41-45) where the mathematical entities of the fractal are generally used with the infinite depth of this structure, despite the empirical inadequacy mentioned above. Smith notes:

> We can live with this, treating it just another case of the way idealizing theories depart from strict truth, if we can find some compensating virtue – roughly, some story about simplicity to trade off against the empirical mismatch. (Smith 1998, 45)

And this simplicity Smith discerns:

> (…) if we stare at the infinite detail of e.g. the Lorenz attractor, we naturally think of it as an astonishingly complex object and then wonder how such a mathematical monster can legitimately get put to empirical work (…). But switch perspectives again, and think of the attractor as what is left fixed in place by a dynamics which stretches and folds phase space trajectories, and we now can see how the needed simplicity might get into the picture. For we could have a dynamic model which specifies relatively simple stretching-and-folding operations, yet (…) even very elementary stretches and folds can have infinitely intricate fractal invariants. (Smith 1998, 46)

My previous depiction of Smith's "new form of idealization" (see Zámečník 2012a, 699-703) now appears to correspond to the concept of abstraction used by Morrison. Similar to her examples, which work with models containing the mathematical infinity entity, we also need an infinite structure of the fractal. It is unavoidable that an explanation of the dynamics of the system is actually present in the form of an infinite intricacy of fractal invariant. The explanatory force of the theory depends on the fact that we work with the mathematical model as abstraction.

3.2. Infinite sets in formal grammar

Mathematical models like abstraction are also found outside the sphere of natural sciences. In linguistics, for example, they manifest themselves in the Chomsky hierarchy of formal grammars, which describes the path to transformational grammar. Even in this case, like Morrison's, we encounter a mathematical infinity, this time in the context of set theory. Again, it is not possible to fully capture the whole theory of the Chomsky hierarchy (see e.g. Partee et al. 1993, 559-561), but only to select the central aspects that will show the role of mathematical models as abstractions.

The fundamentals of Chomsky's transformation grammar are based on automata theory (see Partee et al. 1993, 431-435), when strings generated by individual types of grammars can be identified with strings accepted by individual types of state automata – for example, finite state automata correspond to regular grammars, pushdown automata correspond to context-free grammars and Turing machines correspond to recursive enumerable grammars.

The role of mathematical models as abstractions appears in formal grammars in the very foundations of automata theory, where a crucial role is played by the fact that a power set made up of an infinite set of natural numbers is uncountable. For automata theory, the central aspect of set theory is the fact that one-to-one pairing cannot be done between an uncountable infinite set of real numbers and a countable infinite set of natural numbers.[12] This is because it is impossible to arrange the elements of the set of real numbers in a series, according to the given rules. For example, if we take real numbers from zero to one, we cannot find an algorithm that would lead to an endless series in which all the real numbers from this interval would be successively present (see e.g. Papineau 2012, 30-39).

Given formal grammar as a model of any grammatical system, although this model can be approached as idealization in the sense that formal grammar must be distinguished from the grammar of natural language,[13] formal grammar appears to be a non-reducible abstraction with respect to the above-mentioned aspects of set theory.

Partee states that, given that the means we take into account in the formal grammars for the characterization of language are countable infinite classes, it follows that there is an uncountable infinite number of languages that do not have grammar (in the above sense).[14] Therefore, there are such sets of strings that they cannot be characterized by finite means (see Partee et al. 1993, 433-434). The distinction between individual types of infinities, mathematical models as abstractions, plays a central role in defining the area of formal grammatical descriptions.

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[12] The relationship between these sets is expressed in such a way that each member from the set of real numbers can uniquely pair with a member of the power set of natural numbers. Possibly stronger claims about the nature of the infinity of natural and real numbers are expressed in the continuum hypothesis.

[13] For example, the basic assumption that formal grammar, which is a suitable candidate for the representation of natural language grammar, must be at least slightly context sensitive (see Partee et al. 1993, 501-503).

[14] The argument resides, *in nuce*, on the fact that the language with the dictionary A can be defined as any subset of A^* (see Partee et al. 1993, 433). Assuming that A^* is countable infinite, power set $\wp(A^*)$ is uncountable infinite.

Here we may object to whether it is appropriate to consider abstractions and idealizations in the field of formal grammars if Morrison's and our examples are tied to the natural sciences, whereas here we are basically moving into a formal discipline that fundamentally draws on the set theory and algebra. We believe that this example is relevant and important because the importance of formal grammars rests, among other things, on their modelling role with respect to the natural language grammars (e.g the disputes about context-freeness and context-sensitivity of natural languages, see Pullum & Gazdar 1982, Schieber 1985).

Partee holds that languages characterized by final means show in their strings a pattern that distinguishes them from other strings in A^* (see Partee et al. 1993, 434). Although natural language grammars are much more complex than formal (and therefore we may speak about idealization), it is still essential that we approach natural grammars as sets of rules that simply have to be characterized by finite means. Thus, our concept of the natural language grammar (see also Chomsky's transformational grammar) is bound to work with the abstraction of infinity in the distinction of its countable and uncountable variants.[15]

In automata theory in connection with the Chomsky hierarchy, Turing's machine is of central importance, which accepts the strings generated by unrestricted rewriting systems (type 0 grammar), defining recursively enumerable languages. In concretizing the above, it is true that an infinite number of Turing machines can be uniquely coupled with natural numbers, that is, the Turing machines are countable infinite. Of course, it follows, according to this argument above, that there are uncountable infinite numbers of Turing's unacceptable languages (see e.g. Partee 1993, 505-523).

Morrison does not remain bound by physical examples when she claims that biology needs mathematical models like abstraction (see Morrison 2015, 40). In addition we can say that every comprehensive theory of grammar (not only formal) necessarily requires mathematical models like abstractions.

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15 We are aware that there is a large group of set theory critics with regard to the concept of infinity (see e.g. Vopěnka 1979). This text is intended, inter alia, to provide an apology of the concept of infinity in mathematics.

4. Why we cannot renounce our mathematical abstractions

In the previous two sections, we adhered to Morrison's position advocating the importance of mathematical models as abstractions, not merely as idealizations. We illuminated from a different perspective the role of DST abstractions and we documented that the use of abstractions is not limited by DST and the concept of universality. If we concede that the role of mathematical models is more complex than the pragmatic philosophy of science suggests, then the crucial question arises as to how to elucidate the relationship between mathematics and science.

Morrison puts this question in the above referenced book: "The interesting philosophical question is how we should understand the relation between this abstract structure and the concrete physical systems that this structure purportedly represents" (Morrison 2015, 22-23). This question is about the nature of the relationship between mathematics and physics. The question that Morrison poses elsewhere (see Morrison 2015, 55) is whether it is possible to separate mathematics and physics contained in physical theory.

The discussion in philosophy of science cannot be satisfied with merely spraying individual examples which can support a certain concept of the model. On the other hand, the two newly introduced examples of models designed as abstractions discussed above allowed the Morrison's concept to get rid of its excessive exclusivity in relation to a large but limited set of examples (the renormalization group). At the same time, we have facilitated the redirection of the main emphasis in conceiving abstractions from their role of means of representing phenomena to their role of explanatory theories. We believe that in both examples the binding of mathematical models as abstractions with theoretical principles is obvious (for more see below).

The position to be defended can be illustrated by the argumentation sketch as follows:

1. The inherent role of scientific models is to convey an explanation.
2. Explanation cannot be bound to purely mathematical entities, i.e. a mathematical fact cannot exclusively explain a natural fact.

3. Morrison does not present any concept of abstraction as a mathematical model that allows explanation, which does not contradict point two.
4. A common characteristic of the examples given in the third section is that they contain explanatory model (mathematical abstractions), because of relations of these models to theoretical principles.
5. The unrealistic nature of the model (with respect to point 4) does not prevent the model from participating in the explanation.
6. The concept according to which we define the preceding points is referred to as mathematical conventionalism.

We believe that point one of our argumentation frame does not require a special commentary. It is hard to imagine a science built purely on the base of models as appropriate representations of the system under study, without any possibility of defining their explanatory role. This task is based on the possibility of delimitation of the theoretical principles which the models are based on.[16]

Also, the second point does not need an extensive commentary to be supported, because we probably find only a few authors who would argue with it. Morrison deals with an analysis of several counter-examples, defined by Baker (see Morrison 2015, 50-57), and refuses the Baker's position. We agree with her rejection because we can say in terms of conditional reductionism that all explanations in natural science should ultimately be physical, but when accepting the mathematical explanation of the physical, we might accept the reduction of physics to mathematics.

At point two of our argumentation, it is particularly interesting why Morrison paid such attention. Morrison clearly stands away from a number of concepts of models (primarily she criticizes the concept inspired by Nancy Cartwright), one of the most important being fictionalism (see Morrison 2015, 85-118). We believe that she fails in clear declaration that her approach to mathematical models as abstractions cannot be interpreted just

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[16] I thank to Ladislav Kvasz who once said in a discussion that the concept of models as representations of different systems without the knowledge of any unifying theory recalls the conception of ancient Egyptian science in which no theories existed, but only groups of applicable models/representations.

as a denial of point two. This problem is mainly related to the fact that it is not clear from the Morrison's argument what the physical information content (physical information) carried by the mathematical structure is.

We believe that what Morrison introduces when interpreting abstractions in the context of renormalization theories, i.e. in DST (used in other places as evidence of a specific method of unification in physics, see Morrison 2013), recounts more a sum of formal properties of a mathematical system that can be used to represent a real system. We, thus, believe the concept of Morrison's mathematical models as abstractions is similar to formalism.

In other words, it reminds us of a situation where we would argue that for example differential calculus carries information about the physical system and thus explains a class of dynamic phenomena. This example is pertinent because we also know that the assumption of differential calculus is unrealistic (at least in the context of a discrete structure dictated by quantum physics and a standard model of particles and interactions). But the differential calculus is not almighty, of course, the core of the explanation is ensured, being limiting to classical dynamics, by the Newton's laws of motion.

We claim that Morrison, as pointed out in point three of the argumentation sketch, does not have the tools to actually make models like abstractions able to participate in the explanation without the mathematical structure itself being responsible for the explanation.

The central point of our conception (in point four) is the assertion that what makes models as abstractions explanatory is their association with theoretical principles. Although we do not consider this statement to be controversial (like the one in point one and two), we believe that too little consideration is being given to it in today's professional discussions. Morrison's attempt to use the concept of physical information borne by mathematical abstraction is inadequate because the theoretical principle is an abstract entity that is empirically adequate construction created by a cognitive agent with regard to the unification of phenomena and the comprehensibility of the world. The world is here in agreement with Davidson and Searle (see Searle 2012, 199-200), a regulatory idea that is a condition of the intelligibility of our beliefs.

Smith's definition of the role of fractal geometry in the dynamic systems theory is a piece of evidence of how a mathematical model as abstraction should be conceived in its relation to a theoretical principle. The definition of the attractor of a dynamic system assumes that we have a theoretical principle – in our case it is an abstract entity expressing the strech-fold process of transformation of the phase space. For a special set of dynamic systems, strange attractors can be shown to be empirically adequate models of real systems whose dynamics is in a chaotic mode. And for these cases, it is inevitable to connect the fractal geometry with its infinite structure with an attractor. A mere prefractal would not be an adequate model because it would not express all the essential features of the theoretical principle.

As we have already expressed above in relation to mathematical models as abstractions used in formal linguistics, the basic theoretical principle governing all formal approaches modelling the natural language is the requirement that sets of rules expressing the natural language grammar must be expressed by finite means. This means that when modelling a natural language, we must have a model as an abstraction that distinguishes countable and uncountable infinities.

Beyond the above (in the third section) mentioned, it can be reminded that, as part of generative linguistics built by Chomsky on the basis of the formal grammar hierarchy, we encounter the mathematical model as an abstraction. This model is an embedding operation, which is connected with the basic principle of transformational grammar – with the principle of recursion. The recursive procedure allows you to generate unlimited long strings (sentences) by applying the final set of rules. There is also the need to implement discrete infinity of recursive prescriptions in the model as an abstraction. Also, here the model would not be involved in the explanation if it stated that the number of recursive operations was finite.

In connection with the fifth point of the argumentation sketch, there is the clarification of how the theoretical principle can serve to explain when it has unrealistic properties. We believe that this fifth point is problematic and unacceptable for advocates of most forms of scientific realism. However, since we have already entered constructive empiricism, it is not our intention to refute or otherwise justify the non-adequacy of scientific realism.[17] Because of our rationale that the explanatory force depends on

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[17] As the only consistent form of realism, we admit Searle's external realism, which we interpret transcendentally (see Zámečník 2012b, 25-30).

the relationship between the model and the theoretical principle, we do not have to thematize the issue of realism at all.

The concept of mathematical conventionalism that we stand for in the sixth point of the argumentation is compatible with constructive empiricism (following van Fraassen 2002) and conditional reductionism (following Kim 2005), which we have previously made a part of our argumentation. Constructive empiricism conceives theories (theoretical principles and models) as empirically adequate constructions whose relationship with the world can never be based on isomorphy or, more generally, similarity. Also, as for van Fraassen, we believe that the world is above all a regulatory idea, and that empirical adequacy is defined by empiricism as a stance that prevents some theoretical constructs from being conceptualized as structures (or objects) of reality, hence protect us against metaphysics (see van Fraassen 2002, 36-38).

Conditional reductionism is not necessary to define our conception of mathematical models as abstractions. It states that all explanations should be principally reducible to physical explanation. It is based on the view of physicalism that we can find in Jaegwon Kim, and whose platform is on the concept of functional reduction (assuming physical realization of function) (see Kim 2005, 161-170).

If constructive empiricism refers to the origin and nature of theories (theoretical principles and models), conditional reductionism refers to the principle form of explanation using these theories. We build mathematical conventionalism as a view that expresses the structure and the characteristics of theoretical principles. Mathematical abstractions are the means by which a limited cognitive agent imprints the structure into theoretical principles. Mathematical abstractions (of course, we have taken infinity only, in countable and uncountable forms) are the constructional rules of theoretical principles and hence models. We believe that the origin of mathematical conventionalism can be traced back through van Fraassen (1989) to Cassirer (1923) (and probably to Poincaré).

In science the role of mathematics in modelling is therefore genuinely structural, and we concur with Morrison that this involves both the use of idealizations and abstractions. Pace Morrison, however, we do not believe that the finding of universality (see Morrison 2015, 80-81) implies that the mathematical structure is strictly understood in its explanatory/understanding role independent of chosen theories (working across ontologies).

Morrison's examples chosen from DST obscure the possibility that this mathematical model as an abstraction (e.g. here RG) will be replaced by another, at a given moment, for a given empirical evidence, more appropriate.[18]

Mathematical conventionalism is a position that can be wedged between fictionalism (and formalism) and the transcendent conception of mathematical abstractions in relation to the world. It does not determine a scientific model to the role of useful fiction (or formal description tools) on one hand and of transcendent mathematical entity on the other. Mathematical conventionalism (along with constructive empiricism and conditional reductionism) simultaneously defines the space for the axiology of science, which stands for three fundamental epistemic values: empirical adequacy, unification of theories and the comprehensibility of the world (point-of-view invariance).

5. Conclusion

Here we have striven to demonstrate several examples of DST and to exemplify formal linguistics to support the concept of mathematical models as abstractions as conceived by Margaret Morrison. We have seen that the use of abstractions is not limited to DST. The lack of mathematical models as idealizations, which the utilitarianists favour, does not imply that the central role of mathematical abstraction is a proof of the validity of mathematical Platonism. Abstractions are the necessary equipment of our creation of theories because of the transcendental limits of our reasoning.

Pragmatic orientation in the philosophy of science has seduced us to forget the indispensability of models as abstractions for the creation of scientific theories not only in the fundamental research of theoretical parts of physics, but also in profane and for foreseeable widely applied theories. In conclusion, despite the mainstream, we can say that without mathematical models as abstractions, science would be merely a cataloguing activity. In

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18 See, for example, the versions of physical theories that the need of renormalization understand as the absence of a fundamental theory – a theory that simplifies the expression of unification (see Batterman 2013, 141-188, 224-254).

nuce: scientific hypotheses have an explanatory power in many cases precisely to the extent that the mathematical model is present as abstraction.

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References

BAIN, J. (2013): Effective Field Theories. In: Batterman, R. (ed.): *The Oxford Handbook of Philosophy of Physics*. Oxford: Oxford University Press, 224-254.

BANGU, S. (2013): Symmetry. In: Batterman, R. (ed.): *The Oxford Handbook of Philosophy of Physics.* Oxford: Oxford University Press, 287-317.

BATTERMAN, R. (ed.) (2013): *The Oxford Handbook of Philosophy of Physics*. Oxford: Oxford University Press.

CARTWRIGHT, N. (1983): *How the Laws of Physics Lie*. Oxford: Clarendon Press.

CARTWRIGHT, N. (1999): *The Dappled World: A Study of the Boundaries of Science*. Cambridge: Cambridge University Press.

CASSIRER, E. (1923): *Substance and Function*. Chicago: The Open Court Publishing Company.

CHOMSKY, N. (1957): *Syntactic Structures*. The Hague/Paris: Mouton.

VAN FRAASSEN, B. C. (1989): *Laws and Symmetry*. Oxford: Oxford University Press.

VAN FRAASSEN, B. C. (1989): *The Empirical Stance*. London: Yale University Press.

FRENCH, S. (2014): *The Structure of the World*. Oxford: Oxford University Press.

GELFERT, A. (2016): *How to Do Science with Models: A Philosophical Primer*. Berlin: Springer.

GIERE, R. N. (1988): *Explaining Science: A Cognitive Approach*. Chicago: The University of Chicago Press.

GIERE, R. N. (1999): *Science without Laws*. Chicago: The University of Chicago Press.

GIERE, R. N. (2004): How Models Are Used to Represent Reality. *Philosophy of Science* 71, No. 5, 742-752.

GIERE, R. N. (2006): *Scientific Perspectivism*. Chicago: The University of Chicago Press.

GODFREY-SMITH, P. (2009): Abstractions, Idealizations, and Evolutionary Biology. In: Barberousse, A., Morange, M. & Predeu, T. (eds.): *Mapping the Future of Biology: Evolving Concepts and Theories.* Boston: Springer, 47-56.

KADANOFF, L. P. (2013): Theories of Matter: Infinities and Renormalization. In: Batterman, R. (ed.): *The Oxford Handbook of Philosophy of Physics*. Oxford: Oxford University Press, 141-188.

KELLERT, S. (1993): *In the Wake of Chaos*. Chicago: The University of Chicago Press.

KIM, J. (2005): *Physicalism or Something near Enough*. Princeton: Princeton University Press.

KÖHLER, R. (et al.) (2005): *Quantitative Linguistics. An International Handbook*. Berlin: De Gruyter.

MANDELBROT, B. (1977): *Fractals: Form, Chance and Dimension*. San Francisco: Freeman.

MORRISON, M. (2013): The Unification in Physics. In: Batterman, R. (ed.): *The Oxford Handbook of Philosophy of Physics*. Oxford: Oxford University Press, 381-415.

MORRISON, M. (2015): *Reconstructing Reality: Models, Mathematics, and Simulations.* Oxford: Oxford University Press.

PAPINEAU, D. (2012): *Philosophical Devices.* Oxford: Oxford University Press.

PARTEE, B. H. (et al.) (1993): *Mathematical Methods in Linguistics*. Dordrecht: Kluwer Academic Publishers.

PEITGEN, H.-O. (et al.) (2004): *Chaos and Fractals – New Frontiers of Science*. New York: Springer-Verlag.

PRIGOGINE, I. & STENGERS, I. (1984): *Order out of Chaos: Man's New Dialogue with Nature*. New York: Bantam Books.

PULLUM, G. K. & GAZDAR, G. (1982): Natural Languages and Context-Free Languages. *Linguistics and Philosophy* 4, No. 4, 471-504.

SCHIEBER, S. (1985): Evidence against the Context-Freeness of Natural Languages. *Linguistics and Philosophy* 8, No. 3, 333-343.

SEARLE, J. (2012): Reply to Commentators. *Organon F* 19, supplementary issue No. 2, 199-200.

SMITH, P. (1998a): Approximate Truth and Dynamical Theories. *The British Journal for the Philosophy of Science* 49, No. 2, 253-277.

SMITH, P. (1998b): *Explaining Chaos*. Cambridge: Cambridge University Press.

VOPĚNKA, P. (1979): *Mathematics in the Alternative Set Theory*. Leipzig: Teuber Texte.

WEISBERG, M. (2013): *Simulation and Similarity: Using Models to Understand the World*. Oxford: Oxford University Press.

ZACH, M. (2017): Axel Gelfert: How to Do Science with Models: A Philosophical Primer (book review). *Organon F* 24, No. 4, 546-552.
ZÁMEČNÍK, L. (2012a): Filosofická reflexe teorie chaosu. *Filosofický časopis* 60, No. 5, 685-704.
ZÁMEČNÍK, L. (2012b): External Realism as a Non-Epistemic Thesis. *Organon F* 19, supplementary issue No. 2, 25-30.