SCIENTIFIC LAWS AND SCIENTIFIC EXPLANATIONS: A DIFFERENTIATED TYPOLOGY[1]

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ABSTRACT: The paper tries to provide an alternative to C. G. Hempel's approach to scientific laws and scientific explanation as given in his D-N model. It starts with a brief exposition of the main characteristics of Hempel's approach to deductive explanations based on universal scientific laws and analyzes the problems and paradoxes inherent in this approach. By way of solution, it analyzes the scientific laws and explanations in classical mechanics and then reconstructs the corresponding models of explanation, as well as the types of scientific laws appearing in it.

The paper makes an attempt to provide a new approach to scientific laws and scientific explanations. Based on my paper Hanzel (2007) I give a brief overview of Hempel's approach to scientific laws and scientific explanation, as well as of its failures and paradoxes. As a way out, I analyze the scientific laws and explanations in classical mechanics and then reconstruct the corresponding models of explanation, as well as the types of scientific laws appearing in it. Finally, I provide a differentiated typology of scientific laws and scientific explanations.

KEYWORDS: Hempel, D-N model, gradual concretization, appearances, manifestations.

1. Introduction

The aim of this paper is to provide an alternative to Hempel's approach to one of the core issues of philosophy of science, namely, scientific laws and scientific explanation. I shall compare his view with that of classical mechanics and then, by drawing on the latter, provide a broader view of scientific laws and explanations.

I shall start with a brief list of the main characteristics of Hempel's approach to deductive explanations (the so-called „D-N") based on universal scientific laws and analyze the problems and paradoxes inherent in this approach. By way of solution, I shall analyze the scientific laws and explanations in classical mechanics and then reconstruct the corre-

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sponding models of explanation, as well as the types of scientific laws appearing in it.

2. The D-N model

Restricting my interest to explanations based on universal scientific laws, Hempel's contribution to the issue of scientific explanation can be understood by stating the following list of issues pertaining to his D-N model.[2]

1. Scientific explanation may be construed as an argument in which the explanandum is a logical consequence of the explanans.
2. Hempel in Hempel – Oppenheim (1948) gives only one very superficial example of a real scientific explanation of a particular event (Hempel – Oppenheim 1948, 246) and then, bypassing any detailed analysis of real scientific explanations, proceeds immediately to the explanation of a scientific law, claiming that what was stated for the explanation of a particular event holds also for the case of explanation of a scientific law.
3. The seemingly unproblematic shift from the reconstruction of explanation of a particular event to that of a scientific law accomplished encounters serious trouble when Hempel reconstructs scientific explanation by means of a lower functional calculus in his paper Hempel – Oppenheim (1948). Here he is forced, as stated in Footnote 33 (Hempel – Oppenheim 1948, 273), to restrict his reconstruction only to the case of explanation of a particular event; the D-N model thus fails to reconstruct the case of explanation of a scientific law from other scientific laws.
4. Even if Hempel puts special emphasis on the concept of scientific law, his approach to real scientific laws would nevertheless avoid any attempt at their detailed analysis. Instead he *oversimplifies* its structure and gives its structure as that of the universal conditional form

   (1) $(x)(Fx \rightarrow Gx)$

5. Hempel was forced after nearly a quarter of a century, even with the logically transparent reconstruction (1) of scientific laws, to acknowledge that his approach to scientific laws had failed (Hempel

[2] For a detailed analysis see my paper Hanzel (2007).

1964). This failed approach views scientific laws as displaying the characteristics of being *true*, of being *universal* and as expressing a *general regularity* (Hempel 1942, 231; Hempel – Oppenheim 1948, 247).

6. In addition to the failure of the D-N model to reconstruct the explanation of a scientific law from other scientific laws, the following three fundamental deficits of this model are worth mentioning. (1) The "*causal deficit*," i.e., its inability to reconstruct the causal import of scientific laws. (2) The "*deficit of deduction*." Even if Hempel views scientific explanation based on universal laws exclusively as a *deductive argument*, this approach leads again the inability to distinguish genuine scientific explanations from pseudo-explanations. (3) Its inability to reconstruct what in natural science is taken for granted, namely, to derive various different explananda from the one set of scientific laws. J. Woodward therefore imposes on proposed scientific explanations the *requirement of functional interdependence* not fulfilled by Hempel's reconstruction of deductive explanations based on universal laws; it goes as follows: "(*f*) The law occurring in the explanans of a scientific explanation of some explanandum *E* must be stated in terms of variables or parameters variations in the values of which will permit the derivation of other explananda which are appropriately different from *E*" (Woodward 1979, 46).

3. A Way Out

In order to provide a more sophisticated reconstruction of the structure of scientific laws, compared to Hempel's expressed in (1), let us reconstruct the structure of the law of simple pendulum. The equation $T = 2\pi\sqrt{\frac{l}{g}}$ pertains to a pendulum which meets the following eight idealizations:

1. The force of friction at the fulcrum equals zero, i.e., the decrease of acceleration due to this force equals zero as well.
2. The whole mass of the pendulum is contained in the suspended body, i.e., the acceleration of suspension cord of pendulum equals zero and the acceleration of the whole pendulum is only due to the acceleration of the suspended body.
3. Acceleration due to the action of non-gravitational forces is equal zero because the latter are not acting.

4. The angle of deviation α of the pendulum is so small that it holds $|\alpha| << 1$.
5. Changes of acceleration of the pendulum due to changes of the length l of the suspension cord are equal to zero because l does not change under the influence of the force of gravity.
6. The volume of the suspended body equals zero; it is a mass-point.
7. The movement of the pendulum takes place in a vacuum, i.e., the force of friction of the environment is equal to zero and so is the decrease of acceleration of pendulum due to this force.
8. The acceleration of the pendulum due to the action of forces acting on the physical system where the pendulum is situated is equal to zero because these forces are equal to zero.

Let us represent each of these idealizations as Id_i. The antecedent of the law of simple pendulum involves a conjunction of eight idealizations Id_1 & Id_2 & … & Id_8. If one expresses this conjunction as $Id_{1\text{-}8}$, while $T^{(8)}$ stands for the period given these eight idealizations, "*P*" for a pendulum located in a physical system, $l^{(2)}$ expresses that idealizations Id_2 and Id_5 pertain directly to the length l of the pendulum, and $g^{(6)}$ expresses that idealizations $Id_{1\text{-}3}$, Id_5, Id_7, and Id_8 pertain directly to acceleration g, then the structure of the law of the simple pendulum is as follows:

$$(2) \qquad (x)(Px \;\&\; Id_{1\text{-}8}x \rightarrow T^{(8)}x = 2\pi\sqrt{\frac{l^{(2)}x}{g^{(6)}x}}\,)$$

Since it contains eight idealizations, one can label it as *the law in the eighth degree of idealizations*, or $L^{(8)}$ for short. This law has been stated in the context of physical theories of gravitational as well as non-gravitational forces. $g^{(6)}$ stands here for the acceleration due to the action of the force of gravitation, i.e., it is *the effect of a cause*. The period $T^{(8)}$ in (2) stands for an effect of the force of gravitation acting on a pendulum with a length $l^{(2)}$. So, the law (2), I would argue, is a *causal* type of scientific law.

At the same time the law of simple pendulum with the structure of (2) can serve as the basis of explanation. One can, for example, suppose that the length of the cord *does* change under the impact of the force of gravity from an initial length $l_o^{(1)}$ to the length $l^{(1)}$. This means that one has to abolish idealization Id_6, i.e., it now holds $\neg Id_6$ (non-Id_6), and one has to determine anew the relation between period $T^{(7)}$, length $l^{(1)}$ and acceleration $g^{(6)}$.

Another option is to suppose that the motion of the pendulum takes place under the influence of the resistance of the medium in which the pendulum moves. In this case one has to abolish not only the idealization Id_7 but also to account for the force F_c acting against the force of gravity, so that $F_c = V \cdot d \cdot g$, where V is the volume of the suspended body and d the density of the elements of the environment per unit of volume. Thus, one also has to abolish the sixth idealizations, i.e., it holds $\neg Id_6$. One then obtains from the law with the structure of (2) the following law (where "m" stands here for the mass of the suspended body, "m'" stands here for "$V \cdot d$"):

$$(3) \qquad L^{(6)}: (x)[Px \,\&\, Id_{1\text{-}5,8}x \,\&\, \neg Id_{6,7}x \rightarrow T^{(8)}x = 2\pi\sqrt{\frac{mxl^{(2)}x}{g^{(5)}x(mx - m'x)}}\,]$$

Once one interprets (2), and therefore also (3), as a *causal* type of scientific law, it can be readily seen why Hempel, in his analysis of explanation based on the law for the simple pendulum, ends up with the above reconstructed problem. Even if he views g appearing in the equation $T = 2\pi\sqrt{\frac{l}{g}}$ as the acceleration due to the force of gravitation (i.e., views the law for the simple pendulum as being part of Newtonian mechanics), he does not understand it as a causal type of scientific law but conceives it as a law of coexistence. What is behind this approach is the fact that Hempel draws on a Humean regularity approach to causation and thus to causal laws, as well. So, for example, he characterizes the causal type of scientific law as being "always presupposed by an explanatory statement to the effect that a particular event of a certain kind *G* (e.g., expansion of gas under constant pressure; flow of current in a wire loop) was caused by an event of another kind *F* (e.g., heating of the gas; motion of the loop across the magnetic field). To see this, we need not enter into the complex ramifications of the notion of cause; it is sufficient to note that the general maxim, 'Same cause, same effect,' when applied to such explanatory statements, yields the implied claim that whenever an event of kind *F* occurs, it is accompanied by an event of kind *G*" (Hempel 1966, 53).

The problem Hempel encounters with respect to the law of simple pendulum emerges from the impossibility of distinguishing within the (Humean) regularity approach to scientific causal laws between the equation $T = 2\pi\sqrt{\frac{l}{g}}$ and the equation $l = \frac{Tg}{4\pi^2}$ obtained from the former by a simple mathematical manipulation. Newtonian mechanics, however, *respects the differentiation*. The periods $T^{(8)}$, $T^{(7)}$, $T^{(6)}$ etc. are viewed in it as effects, with the force of gravity acting on the pendulum, as their principal cause. Because Hempel's approach to the law of the simple pendulum rests on a Humean approach to causation and causal types of scientific laws, there is no way for him to distinguish between an equation which expresses a bond of the genesis of an effect from its cause (e.g., the genesis of the period of the swing of the simple pendulum) and the original equation which has been amended and which expresses merely an approach allowing to trace back and calculate the size (quantity) of the cause issuing form the size (quantity) of the effect.

To what scientific laws can the (Humean) regularity approach be applied? In my view it can be applied to those laws where causation is understood as a recurring succession of events of a certain type, so that the cause is understood as an antecedent event and the effect as a consequent event. Let us take on of Hempel's examples of scientific laws, e.g., "Wherever the temperature of gas increases while its pressure remains constant, its volume increases" (Hempel 1965a, 338). Its structure can be represented as follows ("*Gx*" stands for "*x* is a gas", "*Tx* = const" stands for "*x*'s temperature is constant", "*Hx*" stands for "*x* is heated", and "*Ex*" stands for "*x* expands"):

(4) $(x)(Gx \;\&\; Tx = \text{const} \;\&\; Hx \rightarrow Ex)$

Thus, in my reconstruction, contrary to Hempel's superficial reconstruction (1), I take into account i) *the universe of discourse*, i.e., the kind of entities for which the law is stated and ii) *the conditions* under which the entities of this kind undergo iii) certain changes – as *causes*, due to which they undergo iv) other changes as *effects*. The general structure corresponding to (4) can be represented as follows:

(5) $(x)(Nx \;\&\; Cn_{1\text{-}k}x \;\&\; C^{o}x \rightarrow E^{o}x)$,

that is to say, "Whenever and wherever there occur objects of a certain kind *N* under 1 through *k* conditions *Cn* as well as phenomena-events of type C^{o}, then always and without exception phenomena-events of type E^{o} occur." Here "C^{o}" and "E^{o}" stand for a type of cause and a type of effect, respectively, and I label scientific laws with the structure corresponding to that of (4) as *"Humean" type causal law*, or L_{cs} for short. What Hempel labels as the "law of coexistence", or L_{cx} for short, can be reconstructed as follows:

$$(6) \qquad (x)(Nx \,\&\, Cn_{1\text{-}k}x \,\&\, E_1x \rightarrow E_2x),$$

where "E_1" and "E_2" stand for *coexisting* types of phenomena.

If one compares my reconstruction of the law of simple pendulum with its reconstruction by Hempel, one immediately notices the following difference: while Hempel views it as expressing a *non-causal* description of coexisting phenomena-events, I view it as a (*non-Humean*) *type of causal law*. How is such a difference between two reconstructions of seemingly the same scientific law possible? Based on what Hempel states, e.g., in his *Theoretician's Dilemma* (Hempel 1958, 178), it is because the law of simple pendulum he draws on is in fact the law stated before the advent of Newton's laws of motion, of the law of gravitation as well as of other derived laws of Newtonian mechanics. Given the structure of (2), this law *could have at most*[3] the following structure:

$$(7) \qquad (x)(P^*x \,\&\, Id_{1,4,6,7}x \rightarrow T^{(4)}x = 2\pi\sqrt{\frac{l^{(1)}x}{k}}\,).$$

Here "P^*x" stands for "x is a pendulum", while the four idealizations here are obtained by the following reductions: 1) Idealizations Id_2 and Id_5 from (2) drop out because the concept of mass appears for the first time in its modern form only in Newton's *Principia*. 2) Idealization Id_3 drops out of (2) because in the time of Galileo there were no physical theories about non-gravitational forces. 3) k is here just a constant of proportion; it does not stand here for acceleration like g in (2), where acceleration viewed as the effect of the force of gravitation.

Even if the reconstruction of the structure of Galileo's law for the pendulum before the advent of Newtonian mechanics cannot restrict itself to (7) and requires a detailed analysis of the works of G. Galilei and Ch. Huygens, nevertheless the comparison of (2) and (7) shows that the whole structure of the law for the simple pendulum had to undergo a complete restating once it was transformed from a *predecessor to* into an *inherent part of* Newtonian mechanics. Newton's second law of motion

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[3] For an analysis of Galileo's formulation of the law for simple pendulum see, e.g., Naylor (1974a).

serves as one of the mediating links between these two forms of the law for the simple pendulum. Its equation $F = ma$ holds rigorously, if the following two idealizations hold: the accelerated body has a negligible volume, i.e., it is a mass point, and no forces are acting on the physical system were the body is located. Thus, the second dynamic law of Newtonian mechanics has the following form

(8) $(x)(Ox \,\&\, Id_{1,2}x \rightarrow Fx = mxa^{(2)}x)$,

where "O" stands for an object with a non-zero mass occurring in a physical system, "$Id_{1,2}$" stand for the two already stated idealizations and "$a^{(2)}$" stands for acceleration given these two idealizations. Here it becomes readily seen how the law of simple pendulum, initially being a *non-causal law of coexistence* (in Hempel's terminology) is transformed via (8) into a (non-Humean) type of *causal* scientific law. Once one knows the second law of motion then, via the law of gravitation, one can change the equation in (8) into the form of $F = mg^{(2)}$, where $g^{(2)}$ stands for acceleration due to the force of gravitation given those two idealizations, i.e., for *the effect of a cause*. It should now be clear why in (2) I presupposed that the pendulum is in a physical system and why I introduced the idealizations Id_6 and Id_8; they are the result of a conceptual *transposition* from (8) to (2) taking place in the process of derivation of the law for the simple pendulum in the framework of Newtonian mechanics.

A similar conceptual transposition takes place in the derivation of the law of free fall in the framework of Newtonian mechanics. With respect to current physical knowledge it contains the following eight idealizations:[4]

1. The initial velocity of the falling body equals zero.
2. The body falls in a vacuum, i.e., the decrease of acceleration due to the forces of friction equals zero.
3. Non-gravitational forces are not at work; i.e., the acceleration of the falling body due to these forces equals zero.
4. Gravitational forces other than of the central body to which the body falls are not at work, i.e., the acceleration of the falling body due to the action of these other gravitational forces equals zero.
5. The acceleration of the central body due to the action of the force of gravity of the falling body equals zero because the mass of the falling body is much smaller than that of the central body.

[4] I draw here partially on Such (1978).

6. The physical system in which the falling body is placed is free from any acceleration die to the action of some external forces.
7. Acceleration of the falling body is constant at the same distance from the surface of the central body because the force of gravitation of the central body is constant at the same distance from latter.
8. The volume of the falling body is zero; it is a mass point.

The law of free fall can thus be stated as follows:

$$(9) \qquad (x)(Ox \,\&\, Id_{1\text{-}8}x \rightarrow s^{(8)}x = \frac{g^{(6)}xt^{(2)}x}{2}),$$

Here "*O*" stands for the falling object located in a physical system; g appears here as $g^{(6)}$ because idealizations Id_2 through Id_7 pertain directly to it.

The law of free fall (9) can serve as the basis of various explanations. For example one can derive the law for cases when idealization Id_7 does not hold any more. The presupposition that acceleration due to the force of central body is constant at the same distance from the surface of the central body is in fact equivalent to the conjunction of the following three idealizations:[5]

i) The angular velocity of the central body is equal to zero, i.e., it does not revolve around its own axis;
ii) the deformation of the central body is zero, i.e., it is a perfect sphere with a volume of $V = \frac{4}{3}\pi R^3$;
iii) the density of the central body is constant in space, i.e., the distribution of mass in volume V is constant.

If one gradually abolishes these idealizations i) and ii), then one has to take into account changes in the right side of the equation of (9) by introducing the following additional terms:

i) once one abolishes the idealization that the central body does not revolve one has to take into account the angle α determining the latitude at which the body falls via the additional term $-\frac{\cos^2\alpha}{289}$ which takes into account the effect of the centrifugal forces ap-

[5] On this see Such (1978).

pearing in the case of the revolving movement of the central body; and

ii) when abolishing the idealization that the central body is a perfect sphere one has to introduce the additional term – $\frac{98\cos^2\alpha}{55199}$ which accounts for the central body's bulge on its equator due to its revolutions around its own axis. One then ends up with the following law for the law of free fall:

(10) $(x)[Ox \,\&\, Id_{1\text{-}6,8}x \,\&\, \neg Id_7x \rightarrow s^{(7)}x =$

$$\frac{g^{(5)}x(1-\frac{\cos^2\alpha}{289}-\frac{98\cos^2\alpha}{55199})t^{(2)}x}{2}].$$

Let us now try to propose a general scheme for scientific laws, taking into account the structure of scientific laws in Newtonian mechanics as given above as well as a model of explanation corresponding to explanations based on the law of simple pendulum and on the law of free fall.

As seen from these examples Newtonian mechanics is a science which makes it possible to compute the *effects* of a force whose size (quantity) is in turn determined in the second law of motion by means of yet *another effect*, namely, acceleration. The general scheme which would account for the structure of the second dynamic law of Newtonian mechanics is thus as follows:

(11) $(x)\ (Nx \,\&\, Cmod_{1\text{-}k}x = d_{1\text{-}k} \rightarrow f_1(Cx) = E^{(k)}x)$.

I will label it as *the idealized type of law, in the kth degree of idealization, of the cause/ground underlying the phenomenon-effect* $E^{(k)}$, or $L^{(k)}$ for short. Here "N" stands for the type of entities for which the law is stated; "$Cmod_{1\text{-}k} = d_{1\text{-}k}$" stands for a conjunction of 1 through k idealization, namely, that 1 through *k modification conditions are put equal to zero*; "$E^{(k)}$" is the phenomenon-effect in the kth degree of idealization and "$f_1(C)$" stands for a function of a cause defined and identified via its effect. I view scientific laws with a structure corresponding to that of (11) as *scientific laws of the causal type*, but which conceive causation in a manner different from that given in (5) above. In (11) cause is viewed as i) that *what underlies* the phenomenon-effect chosen as the point of departure by means of which one can identify the cause and calculate its size (quantity); and ii) as the *ground* of all those phenomena-effects which can be derived by explanation from (11). As shown above, one can – by identifying the force as the cause underlying acceleration – determine various *other* effects of the acting force: the period of a swinging pendulum, the trajectory of a falling body, the time-effect of a force acting on a body (the moment of force), the path-effect of a force acting on a body (work), etc. Once one derives, in a manner I will reconstruct below, these laws, one can use the latter again for explanations obeying the following general structure.

The point of departure is a type of law with the following structure:

(12) $L^{(l)}: (x)\ [N^*x \ \&\ Cmod_{1\text{-}l}x = d_{1\text{-}l} \rightarrow\ E_2^{(l)}x = f_l(E^{(k)})x]$

Here "N^*" stands for the type of entity for which the law is stated (e.g., falling body, pendulum, etc.); "$Cmod_{1\text{-}l}=d_{1\text{-}l}$" stands for the conjunction of 1 through *l* idealizations; "$E_2^{(l)}$" stands for a phenomenon-effect in the *l*th degree of idealization (e.g., the covered distance, the period of the swing, etc.); and "$E^{(k)}$" stands for the phenomenon-effect by means of which the underlying cause-ground was initially identified and defined in (11). The type of explanation based on (12), as shown above in the examples of explanations based on the law of simple pendulum and the law of free fall, has the following structure: i) one has to abolish the respective idealizations, thus to suppose that now the respective modifications condition are already at work, i.e., that $Cmod_i \neq d_i$ holds; and ii) at the same time take into account the impact of the modification in order to derive the respective phenomena-effects one wants to explain. If one supposes a gradual abolishment of *all* idealizations one obtains the following sequence of types of scientific laws $L^{(k\text{-}1)}, \ldots, L^{(0)}$:

$$L^{(l\text{-}1)}: (x)\ [N^*x \ \&\ Cmod_{1\text{-}(l\text{-}1)}x = d_{1\text{-}(l\text{-}1)} \ \&\ Cmod_l x \neq d_l \rightarrow E_2^{(l\text{-}1)}x = f_{l\text{-}1}(E^{(k)}x,\ Cmod_l x)]$$

$$L^{(0)}: (x)\ [N^*x \ \&\ Cmod_{1\text{-}l}x \neq d_{1\text{-}l} \rightarrow\ E_2^{(0)}x = f_0(E^{(k)}x,\ Cmod_l x, \ldots, Cmod_1 x)].$$

In the antecedent of $L^{(l\text{-}1)}$ the expression "$Cmod_{1\text{-}(l\text{-}1)}=d_{1\text{-}(l\text{-}1)}$" expresses that 1 through (*l*-1) idealizations are still valid, while the inclusion of "$Cmod_l$" on the right side of the equation expresses that the impact of the *l*th modification conditions is already being taken into account. The type of explanation by means of which one derives $L^{(l\text{-}1)}, \ldots, L^{(0)}$ from $L^{(l)}$, I label – following Nowak (1972) and Such (1978) – as *explanation by gradual concretization*.

Let me now compare the reconstruction of the method of explanation by gradual concretization with Hempel's D-N model as well as my reconstruction of the law of the cause/ground $L^{(k)}$ with Hempel's reconstruction of scientific causal laws.

First, the method of explanation by gradual concretization reconstructs, contrary to the D-N model, the case of explanation of scientific laws from other scientific laws. This case can be represented symbolically as follows ("$-_{c}-|$" stands here for gradual concretization, "$Cmod_i$" for the introduction of a modification condition, i.e., for "$Cmod_i \neq d_i$"):

$$(13) \quad L^{(l)} \;\&\; Cmod_l \;-_{c}-|\; L^{(l-1)} \;\&\; Cmod_{l-1} \;-_{c}-|\; L^{(l-1)} \;\&\; \ldots \;-_{c}-|\; L^{(1)} \;\&\; Cmod_1 \;-_{c}-|\; L^{(0)}$$

The law of the type $L^{(0)}$, i.e., where all idealizations have already been abolished can then serve as the basis of explanation by means of the introduction of 1 through *s* singular conditions $Csin_{1\text{-}s}$. In such an explanatory step one can explain a singular phenomenon $E^{(0)}a$. The whole sequence of explanations then is as follows ("$\supset$" stands here for logical consequence)

$$(14) \quad L^{(l)} \;\&\; Cmod_l \;-_{c}-|\; L^{(l-1)} \;\&\; \ldots \;-_{c}-|\; L^{(0)} \;\&\; Csin_{1\text{-}s} \supset E^{(0)}a$$

The fact that the model of explanation by gradual concretization is capable to reconstruct the case of explanation of a law from other laws is based on an enlargement of the conceptual framework upon which the concept of scientific law is built. In addition to the concept of singular conditions it introduces the concept of *modification conditions*. The latter are causally relevant for whole classes of phenomena-effects and are stated *explicitly*, initially in the form of idealizations, *inside the structure of scientific laws*. *Singular conditions*, on the contrary, are relevant for a respective singular phenomenon and are not stated explicitly inside the structure of scientific laws.

Second, the model of gradual concretization fulfills J. Woodward's requirement of functional interdependence (*f*). To show this let us take the law of the type $L^{(l)}$ given in (12). Till now, for the sake of simplicity, I took into account only *one sequence* of gradual abolishment of idealizations, namely, $Cmod_l \neq d_l$, $Cmod_{l-1} \neq d_{l-1}$, …, $Cmod_1 \neq d_1$, and therefore also only *one sequence* of less and less idealized scientific laws. But in fact one can by means of the method of gradual concretization derive several different sequences of less and less idealized laws depending on which idealization will be abolished as the first one, as the second one, as the third one, etc. from the set of all idealizations. One can thus obtain the following network of scientific laws (the lower index indicates which of the *l* idealizations has already been abolished; "⊥" stands here for gradual concretization).

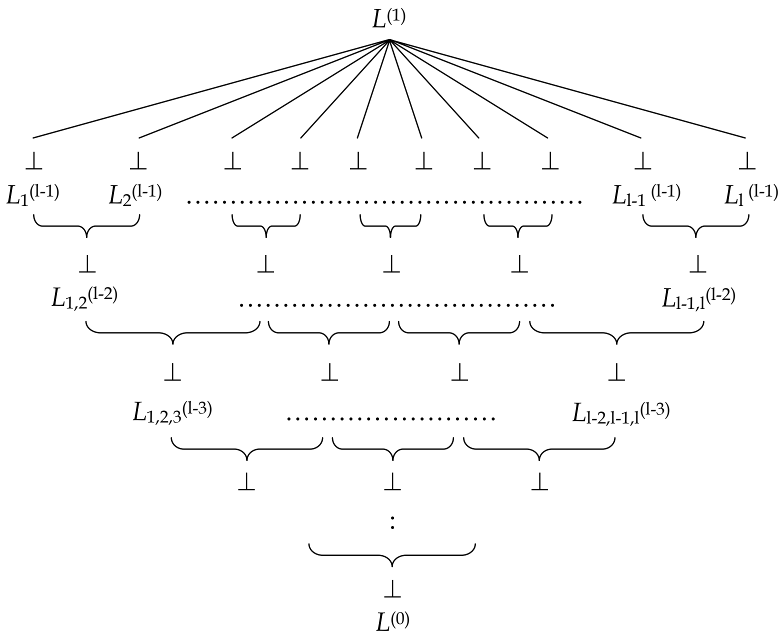

Fig. 1. Network of scientific laws obtained by gradual concretization

It is now readily seen that explanation by gradual concretization based on idealized laws of the type $L^{(l)}$ fulfills the (*f*)-requirement because it holds *that the more a scientific law of this type contains idealizations, the more different explananda (scientific laws and/or explanations of singular phenomena) can potentially be derived from that scientific law once it appears in an explanans* – I emphasize *potentially* – because each concretization step involves *an irreducibly heuristic moment*, namely, *the discovery of the type of causal impact of the respective modification condition*; in explanation by gradual concretization the explanandum is not a logical consequence of the explanans.

Third, while the D-N model views scientific explanation based on universal laws as a process where one explains a phenomenon by i) subsuming it under a (covering) law, and then ii) deduce it from the laws and singular conditions, in explanations of a singular phenomenon-effect represented by (14), e.g., of a concrete value of the period of swing of a real pendulum, say, in a laboratory, one has i) to subsume the singular phenomenon of a certain type to be explained *not under a (covering) scientific law, but only a covering kind N* and the respective covering cause C grasped via a function of the phenomenon* $E^{(k)}$, both appearing in a law of the type $L^{(l)}$, then ii) gradually concretize the law of the type $L^{(l)}$ and, only then, iii) bring in the respective singular conditions for the deduction of the singular phenomenon to be explained.

Fourth, by the reconstruction of the idealized laws of mechanics, by the reconstruction of the idealized law of the types $L^{(k)}$ and $L^{(l)}$, and by providing the reconstruction of the method of explanation by gradual concretization one at the same time can overcome the one-sided logico-normative orientation of Hempel's approach to the reconstruction of scientific laws and explanation. The terms appearing in my reconstructions of laws of the types $L^{(k)}$ and $L^{(l)}$ are based on an analysis of the laws of Newtonian mechanics; they show that it is possible to deal with the concept of scientific law by drawing on the real practice of science. My reconstruction shows also that in order to overcome the problems encountered by the D-N model one has to analyze and reconstruct in detail the respective laws and explanations based on them as they are given in science. With respect to this it is now clear how one should eliminate the "causal deficit" of the D-N model. One has to analyze what does the respective physical theory state – *if it states anything at all* – about the production of the shadow. For example the law of pendulum stated by Galileo, tentatively reconstructing above in (7), does not state anything about the cause of the period of the swings.[6]

Fifth, my reconstruction of the laws of the types L_{cs}, L_{cx}, $L^{(k)}$ and $L^{(l)}$ shows that what unifies them, independently of whatever typological differences there are between them, is the fact that they are *always stated only in respect to entities of a certain kind*. Hempel's attempt to frame the concept of fundamental scientific law via the condition that the latter should hold for all times and spaces is thus a product of a superficial reconstruction of the structure of scientific laws which does not take into account that fact.

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6 For analysis of phenomenological laws see, e.g., Bunge (1964).

Sixth, as shown above, one has in fact *two* laws of simple pendulum: one which antedates Newtonian mechanics and another which is derived in the framework of the latter. The same "double" existence can be identified also for the law of free fall. By drawing on Galileo's *Two New Sciences* (Galilei 1974) and on the reconstructions of the historians of science,[7] the law of free fall as stated by Galileo could have the following structure:

(15) $(x)(O'x \,\&\, Id_1x \rightarrow s^{(1)}x \propto t^2x)$,

where "O'" stands for an object falling with a uniformly accelerated motion from the state of rest (i.e., its initial velocity is equal to zero); "Id_1" stands for the idealization that the fall of the body occurs in vacuum; "$s^{(1)}$" expresses distance covered by the body under this idealization; "t" stands for the time in which it covers that distance and "$\propto$" is the sign of proportionality. (15) can be viewed as a (Humean) regularity type of scientific causal law with a structure corresponding to that of (5). It states: Once a uniformly accelerated body is released from rest (type of a phenomenon as a type of a *cause*) in vacuum, it will cover a certain distance (type of phenomenon as a type of an *effect*) proportional to the square of time in which falls. The "double" existence of the laws of physics suggests that in the development of science *types of phenomena* appearing initially in the (Humean) regularity types of scientific laws – the latter being symbolized above as L_{cs} and as L_{cx} – are *reinterpreted* on the basis of laws of types $L^{(k)}$ and $L^{(l)}$. This typological reinterpretation thus suggests a conceptual differentiation, in the framework of the philosophy of science, between *two types of phenomena*. Phenomena which were initially identified and cognized *prior* to the identification and discovery of their underlying cause/ground, including that which enables to determine the size (quantity) of the latter, I label *phenomena as appearances*; phenomena which are derived from a cause/ground I label *phenomena as manifestations*. Such a change of the phenomena as appearances E_1, E_2, …, E_n, via the cause/ground C – the latter being identified via an appearance in the kth degree of idealization $E^{(k)}$ – into phenomena as manifestations $E^{(l)}{}_1$, $E^{(l-1)}{}_2$, …, $E^{(l-j)}{}_r$ can be symbolized as follows ("df" stands here for identification and definition):

[7] See, e.g., Drake (1973) and Naylor (1974b).

$E_1, E_2, \ldots\ldots, E_n$	appearances
$\downarrow$	
$f_1(C) =_{\mathrm{df}} E^{(k)}$	underlying cause/ground C identified via the idealized appearance $E^{(k)}$
$\swarrow \quad \searrow$	
$E^{(l)}{}_1, E^{(l-1)}{}_2, \ldots\ldots, E^{(l-j)}{}_n$	manifestations

Fig. 2. Conversion of appearances into manifestations

Once one differentiates conceptually between phenomena as appearances and phenomena as manifestations of a cause/ground, one also has to differentiate conceptually between two types of scientific laws pertaining to phenomena: *laws of appearances* and *laws of manifestations*. So, e.g., while the law of free fall and the law of simple pendulum as stated by Galileo are of the former type, the law of free fall and the law of simple pendulum derived in the framework of Newtonian mechanics are of the latter type.

In order to make my conceptual differentiation as precise as possible I differentiate further between *general appearances* – stated in the framework of laws of appearance – and *singular appearances* which one can deductively derive from these laws together with the respective singular conditions. Finally, I differentiate also between *general manifestations* – stated in the framework of laws of manifestations – and *singular manifestations*, the latter being derived by gradual concretizations *and* deduction explanations from these laws together with the respective modification and singular conditions. This means that the epistemic status, e.g., of a concrete value of the swing of an individual pendulum explained on the basis of the law for the simple pendulum as stated by Galileo, and tentatively reconstructed in (15), differs from the epistemic status of a concrete value of the swing of an individual pendulum derived by gradual concretization *and* deduction from the law of the simple pendulum (2) as stated in the framework of Newtonian mechanics. The former concrete value has the status of an *individual ap-*

pearance, the latter concrete value that of *an individual manifestation*. I thus enlarge figure 2 as follows:[8]

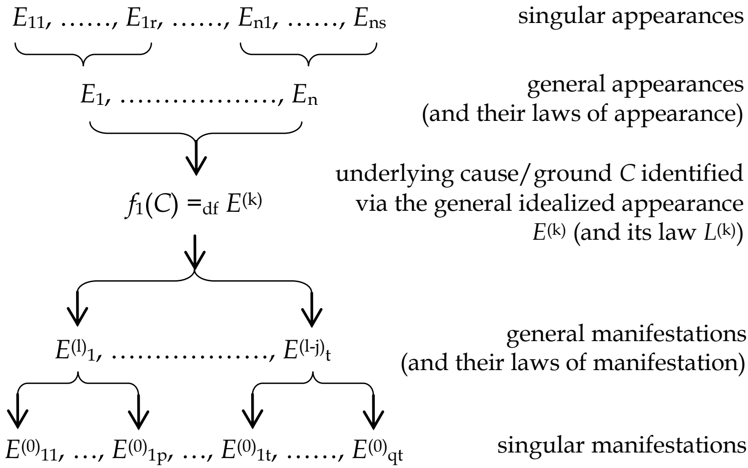

Fig. 3. Change of appearances into manifestations via the underlying cause

I can now give a final evaluation of Hempel's D-N model. It fits explanations based universal scientific laws which have the status of a *law of appearance*, or, as he labels them, that of an *empirical law* and which "asserts a uniform connection between different empirical phenomena or between different aspects of an empirical phenomenon. It is a statement to the effect that whenever and wherever conditions of a specific kind *F* occur, then so will always and without exception certain conditions of another kind, *G*" (Hempel 1966, 54). Even if these laws very often contain idealizations, as shown above in the case of the law of free fall stated by Galileo, explanations based on empirical types of scientific laws can still be viewed as having the nature of a subsumption under these (covering) laws plus deduction; of course, under the supposition that one is not forced to abolish idealizations in the course of explanation. So, e.g., if

[8] In order not to overburden this figure I do not place the arrows at the very top to indicate the epistemic relation between the singular appearances, the general appearances with their laws of appearances. I view the singular appearances as the *presupposition* for the statement of the general appearances together with their laws of appearances and as *the explanatory consequence* of the laws of appearances.

Hempel states the law "Below 32°F, under normal atmospheric pressure, water freezes" (Hempel 1942, 232), then one can derive by deduction from this law and the singular condition expressed as "This sample of water had a temperature below 32°F and was hold at normal atmospheric pressure", the explanandum-statement "This sample of water froze".

Hempel's D-N model as well as his regularity approach to causal types of scientific laws becomes however insufficient as an instrument for a philosophical analysis and explication of the concepts of scientific law and explanation once explanation involves i) the abolishment of idealizations stated in the structure of the scientific law, and ii) the derivation of phenomena as manifestations based on causal laws identifying an underlying cause/ground.

Worth to be mentioned here is the fact that while Hempel's works give a quite complete reconstruction of the deductive explanation based on universal laws of appearance (or, in his terminology, universal "empirical laws"), one can find in them also some *hints* at the existence, but *never a reconstruction*, of explanations based on laws identifying the causes underlying the phenomena, where these phenomena are initially described in universal "empirical" laws. So, e.g., he states that "[t]heories are normally constructed only when prior research in a given field has yielded a body of knowledge that includes empirical generalizations or putative laws concerning the phenomena under study. A theory then aims at providing a deeper understanding by construing those phenomena as manifestations of certain underlying process governed by laws which account for the uniformities previously studied, and which, as a rule, yield corrections and refinements of the putative laws by means of which those uniformities have been previously characterized" (Hempel 1970, 142).

The conclusion I thus arrive at is that Hempel's reconstruction of deductive explanation based on universal laws of appearance is *not invalid but valid, even if only for that type of sciences which access their objects initially only as appearances*. Explanations based on idealized laws with the structure of (11) and (12) are performed in those type of sciences where one can already transform the laws of appearance into laws of manifestation based on the grasping of the underlying cause. The method of explanation I labeled as "gradual concretization" is based, as shown above, on the subsumption of a phenomenon-effect to be explained under a covering universe of discourse (i.e., under a certain kind of entities) and the corresponding cause/ground. From this one can immediately realize *what are the limits of the method of explanation by gradual concretization*. If one wants to explain a phenomenon-effect that belongs to a universe of discourse (a type of entity) which is different from that stated in the respective idealized law of the underlying cause, then the method of explanation by gradual concretization cannot be employed any more, e.g., as in the case when one wants to explain the distance covered by a block sliding on an inclined plane starting from the second dynamic law in Newtonian mechanics. The former pertains to a *block sliding on an inclined plane*, the latter, however, to *a mass-point given in a physical system*.

To widen my typology of explanations based on universal scientific laws let us briefly reconstruct how Newtonian mechanics deals with the derivation of the law for the distance covered by a block sliding on an inclined plane. Starting from the second dynamic law and the law of gravity it performs a *thought-reconstruction* so that the sliding block becomes a mass-point in a physical system sliding down the inclined plane, i.e., an idealized thought-object and then states for this type of entity the equation $\frac{m\mathrm{d}v}{\mathrm{d}t} = mg\,sin\alpha$, where the rightside expression stands for the component of the force of gravity acting on the mass- point sliding down an inclined plane. From this equation, together with its universe of discourse and the stated idealizations, it is then possible to derive the following law $L^{(4)}$ for the distance covered by the sliding mass-point:

$$(16)\quad (x)(O''x \;\&\; Cmod_{1,2,3,4}x = d_{1,2,3,4} \rightarrow s^{(4)}x = g^{(3)}xt^2x\,sin\,\alpha x),$$

where 'O''' stands for the mass-point in physical system sliding on an inclined plane; "$Cmod_{1,2} = d_{1,2}$" stands for the conjunction of the two idealizations given already in the second dynamic law, namely, that the considered object has a zero volume (i.e., it is a mass-point) and that it is placed into a physical system free from any impact of external forces; "$Cmod_3 = d_3$" stands for the idealization that the sliding mass-point starts its sliding from rest (i.e. its initial velocity is equal to zero); "$Cmod_4 = d_4$" stands for the idealization that there is no force of friction decreasing the accelerated motion of the mass-point along the inclined plane; "$g^{(3)}$" stands for acceleration due to the force of gravitation involving already two idealizations transposed from the second dynamic law of Newtonian mechanics reconstructed above in (8).

Based on (16) one can explain, by abolishing idealization $Cmod_4 = d_4$, how the covered distance changes once the force of friction is at work. From $L^{(4)}$ one obtains by gradual concretization $L^{(3)}$ (the force of friction is proportional to $mgcos\alpha$)

$$(17) \quad (x)[O''x \; \& \; Cmod_{1,2,3}x = d_{1,2,3} \; \& \; Cmod_4\, x \neq d_4 \rightarrow s^{(3)}x = g^{(2)}xt^2x(sin\alpha x - cos\alpha x)].$$

(16) and (17) can be viewed as laws of the manifestation of an underlying cause; in the former the covered distance is the effect of the force of gravity and in the latter it is the combined effect of the force of gravity and of the force of friction.

From the example of the derivation of the law for the distance covered by a block sliding along an inclined plane it is clear that the explanatory move from a law of the type $L^{(k)}$ to laws of the type $L^{(l)}$ involves a *thought-reconstruction of a situation one wants to explain by laws of the latter type in such a way that the this situation after the thought-reconstruction fits already the universe of discourse of the law of the type* $L^{(k)}$. What of course unifies the law of the type $L^{(k)}$ with the laws of the type $L^{(l)}$ is *that in all of them one and the same main underlying cause/ground is presupposed*. If the cause/ground given in the explanans-law and the cause/ground in explanandum law are different, then neither the method of thought-reconstruction nor that of gradual concretization can be applied. These two methods of explanation, like explanation based on laws of appearance, have a certain range of application beyond which they cannot be used any more.

With this limitation I can now summarize my differentiated approach to the typology of universal scientific laws and explanations based on them as follows:

The epistemic status of phenomena in the laws of the explanans	*The corresponding type of scientific law in the explanans*	*The corresponding type of conditions in the explanans*
general appearances	law of appearance	singular conditions
idealized general appearance and idealized general manifestations	idealized law of cause underlying the idealized appearance and laws of manifestation	modification and singular conditions

The corresponding type of relation between the explanans and explanandum	*The corresponding type of explanandum*
deduction	singular appearances
construction of idealized objects, gradual concretization and deduction	laws of manifestation and singular manifestations

Fig. 4. A typology of scientific laws and scientific explanations

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