

# Frege on Knowing and Individuating Senses

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**ABSTRACT:** This paper discusses Frege's views on how Fregean senses, *Sinne*, should be individuated, and what chances we have of arriving at knowledge about them. There is a conflict in Frege's views. First he introduces a criterion of sameness of sense which requires that speakers are authoritative concerning senses, then holds that there are cases where no one knows the sense of an expression. If no speaker is authoritative concerning sense, then the original criterion for sameness of sense cannot be upheld. But Frege repeatedly emphasized the need for criteria of identity. This paper discusses the conflict between the different things Frege wants to say about sense, and points to possible ways out of the problems.

**KEYWORDS:** Frege – identity – mode of presentation – sense.

## 1. Introduction

We can find a conflict in Frege's characterization of sense.<sup>1</sup> There should be a tension between on the one hand the criterion of identity Frege proposes for senses, and on the other hand his view that there are cases where no speaker knows the sense of a given expression. It is a conflict which should have been obvious to Frege, but it seems that he never discussed it, or even appeared to view it as a problem. This lack of interest should tell us something about how to understand Frege's notion of sense. I will indicate the strands in Frege's thinking that lead up to the apparent

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<sup>1</sup> I use the English terms "sense" and "reference" for Frege's German "Sinn" and "Bedeutung", respectively.

conflict, without arguing for the ideas behind them. Then I say something about how the conflict should be resolved, and what consequences this resolution can have for our understanding of Frege's notion of sense. This way of resolving the puzzle will then also show something about how Frege thought that we can arrive at knowledge of sense. Some difficulties remain for the proposed solution, but this proposal appears to retain most of what is central in Frege's work. Some tension will remain in Frege's views on any account of the matter. In the last section, I will very briefly sketch a kind of position concerning our knowledge of abstract objects which can be seen in Frege's work, a position that could be called "rationalist pragmatism". Tyler Burge has been emphasizing the rationalist pragmatism of Frege in his writings on Frege (now collected in Burge 2005), and the sketch I provide is no more than a sketch; developing this sketch is properly a subject for another paper.

The dominant interpretation of Frege's notion of sense has been to see it as something related to language, something like linguistic meaning, except for a few more or less supposedly peripheral cases, such as vague expressions, proper names and indexical expressions. It is also a notion tied to *understanding*. A person who understands a sentence is, according to this interpretation, said to do this in virtue of having grasped the sentence's sense, the thought expressed. Then this notion is used as the starting point for constructing a theory of meaning for a language.

Even if this interpretation is historically important, and clearly captures much of importance in Frege's thinking, it has always been clear that it is problematic. Frege's at times almost contemptuous attitudes towards ordinary language,<sup>2</sup> as it is used in communication, and the extent of the idealization of the kind of language he was interested in, have been toned down or neglected in this interpretation, and Frege's repeated claims that his intention was to say something about the structure of *thought*, not language, have been played down.

This interest in thought was prominent in Frege's work from the very beginning. In *Begriffsschrift* (Frege 1879), Frege characterized his goals in the following manner:

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<sup>2</sup> As for instance in "A person who wants to learn logic from language is like an adult who wants to learn how to think from a child" (Frege 1980b, 41).

I did not want to present an abstract logic in formulas, but rather display a content by means of written signs in a manner that was more precise and more surveyable than is possible by the use of words. (Frege 1879, 97)<sup>3</sup>

*Begriffsschrift* had the subtitle *A formula language, modeled upon that of arithmetic, for pure thought*. In an early paper, Frege quotes Leibniz with approval: “A *lingua characterica* ought, as Leibniz says, *peindre non pas les mots, mais les pensées*” (Frege 1880/1881, 13).<sup>4</sup> Of course, these two passages antedate the distinction between sense and reference, but there are also later passages, where it seems that Frege sees the need for a linguistic clothing of thoughts as something which distracts from his real interests. A late passage that highlights Frege’s views on the relations between logic and language is from 1915:

If our language were logically more perfect, we would perhaps have no further need of logic, or we might read it off from the language. But we are far from being in such a position. Work in logic just is, to a large extent, a struggle with the logical defects of language, and yet language remains for us an indispensable tool. Only after our logical work has been completed shall we possess a more perfect instrument. (Frege 1915, 252)

Language is at times inappropriate for the true expression of thoughts, as is the case with expressions for functions (Frege 1914, 239). Footnote 4 of “Thoughts” says:

I am not here in the happy position of a mineralogist who shows his audience a rock-crystal: I cannot put a thought in the hands of my readers with the request that they should examine it from all sides. Something in itself not perceptible by sense, the thought, is presented to the reader—and I must be content with that—wrapped up in a perceptible linguistic form. The pictorial aspect of language presents difficulties. The sensible always breaks in and makes expressions pictorial and so improper. So one fights against language, and I am compelled to occupy myself with language although it is not my proper concern here. (Frege 1918a, 13)

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<sup>3</sup> See also *ibid.*, VI-VII.

<sup>4</sup> Leibniz’s French can be translated “not depict words, but thoughts”.

The idea behind the language-oriented interpretation has been that sense still could function as the main concept for a theory of meaning as a theory of understanding, even if we make room for the things Frege said about the shortcomings of natural language. There are good reasons for seeing sense as intimately connected with language. Dummett has often stressed that there is no other good explanation of what sense is, unless through bringing in some linguistic means for expressing these thoughts (for instance in Dummett 1991, chapter 12). The conflict I will be discussing below can be seen as a special case of the tension in Frege's thought between seeing sense as tied to the realm of thought, and seeing it as essentially connected with language, and thus that differences in sense are essentially tied to their linguistic expressions. In the next section, I will sketch how Frege introduces senses. The following section discusses how sameness of sense is to be determined, and the remaining sections describe how Frege's views on sense and sameness of sense lead to a conflict, and how this conflict is to be resolved.

## 2. Frege's introduction of sense

Frege's notion of sense is usually discussed as if the most important aspect of this notion were the introduction of sense for singular terms, proper names, and that the problem of informative identities—"Hesperus = Phosphorus"—was the central reason for introducing the distinction between sense and reference. This is historically not quite correct. Frege had, almost *en passant*, introduced that distinction in "On Function and Concept" (Frege 1891a), published before "On Sense and Meaning" (Frege 1892), although the two papers were written more or less at the same time. In "On Sense and Meaning", the claim that sentences have a *Bedeutung* was of equal importance to Frege, which can be seen from the space he allots to defending the latter claim. Beaney argues persuasively that part of the need for introducing a distinction between sense and reference stems from Frege's needs to justify aspects of his logicist project, and in particular to explain why the logicist definitions of central mathematical terms, such as "number", should be understood in precisely the manner suggested by Frege (Beaney 1996, chs. 5–8). This rationale for introducing a distinction between sense and reference will not make the introduction of sense for singular terms *the* reason for having a sense-reference distinction. I will

still, however, concentrate upon the senses of singular terms, partly for expository reasons, partly because the problem at hand is more salient there. The question for Frege is then how “Hesperus = Phosphorus” can differ from “Hesperus = Hesperus”, how they can be of different value for a subject. Frege’s answer is that the two names, “Hesperus” and “Phosphorus”, have different senses, because their referent, Venus, is presented in two different ways. The first explanation of sense is then that sense is that wherein the *mode of presentation* is contained (Frege 1892, 57/27). Frege does not straightforwardly *identify* sense with the mode of presentation of the object: all he says is that the mode of presentation is contained in the sense. It is not entirely clear what the difference might amount to. One possible interpretation is that mode of presentation is a more inclusive category, seen as being of relevance for perception as well, since there could be informative identities for a subject who has devised no linguistic means of thinking of an object, whereas sense might be confined to that for which there is a linguistic expression. Another important difference between sense, as conceived by Frege, and modes of presentation as naturally understood, is that Fregean sense is to some extent independent of how the reference is actually apprehended, whereas modes of presentation are naturally understood precisely as features of the *actual* apprehension of the reference. But since Frege is introducing a technical term here, we should perhaps not try to strain the interpretations by reading too much into the notions from our preconceptions.

The sense of a proper name is grasped by everyone who is sufficiently familiar with the language (Frege 1892, 57/27). Sense is something which is both generally cognitive and tied with the understanding of the language.

### 3. The intuitive criterion of sameness of sense

Criteria of identity are required for senses. This is not just a lesson from Quine; Frege could have subscribed to the “No entity without identity” slogan. In §62 of *The Foundations of Arithmetic* (Frege 1884), Frege stresses the need for criteria of identity:

If we are to use the symbol *a* to signify an object, we must have a criterion for deciding in all cases whether *b* is the same as *a*, even if it is not always in our power to apply this criterion. (Frege 1884, 73)

If senses are to be accepted, there should be some criterion of identity for them. In accordance with the way in which sense has been introduced, and the point of having them, Frege provides a criterion of identity for senses.

The intuitive criterion: If it is possible for a speaker to rationally have different cognitive attitudes towards two sentences  $S$  and  $S'$ , then there is a difference in sense.

I have followed Evans in calling this the “intuitive” criterion, since it makes the subject’s judgments of sameness and difference in cognitive attitudes that decide sameness and difference in senses.<sup>5</sup> This criterion is well entrenched in Frege’s thought. It is used implicitly in “On Sense and Meaning” (see the opening pages of Frege 1892), and it is formulated explicitly in letters to Husserl and Jourdain, so it is found from 1891 to 1914. It is also put to use in “Thoughts”, in 1918. A “difference in cognitive value” means that the speaker thinks that the two sentences can differ in truth value. A relevant passage is the following, from a letter to Jourdain, in 1914:

What is expressed in the sentence “Ateb is Afla” is not at all the same as the content of the sentence “Ateb is Ateb”.... In accordance with this, the sense of the sentence “Ateb is at least 5000 meters high” is different from the sense of the sentence “Afla is 5000 meters high”. Someone who holds the latter to be true, is in no way forced to hold the former to be true. (Frege 1980b, 112)

For Frege, there is a normative aspect to this criterion; it is not just a question of what actual speakers actually do, when confronted with appropriate pairs of sentences, it is a question of what attitudes are rational for a speaker who knows the language. What is required for being rational or knowing the language is not spelled out in any great detail by Frege; it seems that he had a common-sense notion of this in mind. An intuitive criterion along these lines is not far-fetched, since sense for singular terms is introduced precisely in order to account for the possibility that a speaker thinks that two sentences differ in truth value, even if the only difference between them is that different terms, referring to the same things, are used in these sentences. The introduction of a notion of sense can then contri-

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<sup>5</sup> See Evans (1982, 18-19).

bute towards a construction of a theory of meaning that can serve to explain the activities of speakers, a project that was not Frege's but still is attractive.

Sense is thus introduced to account for the possibility of subject's rationally holding different attitudes towards sentences that differ only in the way something is thought about, or presented to the mind. If sense is to capture the possibility of speakers' rationally entertaining different attitudes towards such sentences, then sense would have to be exactly as finely individuated as it is possible for a subject to have the different attitudes and still remain rational. If this were not the case, the subject might hold different attitudes concerning the same sense, and the introduction of sense would not serve its purpose.<sup>6</sup> The intuitive criterion will then mean that the subject who has different attitudes towards different sentences must be strongly authoritative concerning her own attitudes and what senses she is grasping. If it were unclear to the subject which sense she were grasping, then the intuitive criterion could not be made to work in the way intended. So sense must be transparent to the subject. Dummett has formulated the requirement of transparency in the following way: "(A) sense cannot have any features not discernible by reflection on or deduction from what is involved in expressing it or in grasping it." (Dummett 1981b, 50) There are, however, factors that complicate this picture. One is that even if Dummett's gloss on transparency is accepted, there is much room for uncertainty—what makes a feature discernible by reflection or deduction? Is there, for instance, some upper limit on how much reflection is needed?<sup>7</sup>

There can be genuine uncertainty concerning what does follow when we reflect on the properties of a given concept. All the deductive consequences of an axiom are rarely available when we start thinking about something. The results concerning the properties of continuous functions that were obtained in the nineteenth century could have served as an inspiration for Frege's thought here. It took a great deal of effort from clever people to sort out what a continuous function was, and what was excluded by being a continuous function.

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<sup>6</sup> Or we might find a need to introduce a new level of metasenses, meant to explain why a subject rationally could hold different attitudes towards one and the same sense.

<sup>7</sup> There has been a renewed interest in transparency lately, as for instance in Fine (2007, especially 60–64), and Sainsbury and Tye (2012). Aspects of these recent views are discussed in Stjernberg (forthcoming).

#### 4. Do we know senses?

The realist, or Platonist, aspects of Frege's thought are well-known. Frege repeatedly insisted that there was a vast difference between being true and being held to be true. Abstract entities like senses, thoughts, concepts and numbers are imperceptible, and exist outside of time and space, independently of us and our possibility to gain knowledge about them.<sup>8</sup> These views of Frege are outside the scope of this paper, but we can note that Frege's kind of realism about abstract entities creates difficulties for the employment of the intuitive criterion: if the existence of senses really is independent of us and our knowledge, why should we then be credited with privileged authority in our judgements about the identity and diversity of sense? Just postulating such an ability doesn't help. There are some options for handling this problem.

One is to reformulate the intuitive criterion as a hypothetical criterion, possible to use if the subject does have a clear grasp of the relevant senses. Frege may have had something like this in mind (since the setting of his presentations of the intuitive criterion demand that the speaker *understands* the expressions).<sup>9</sup> The problem with this suggestion is that it requires that there is a way to know whether a subject does have this clear grasp, and it seems that the only way to do this would be to have some other criterion of identity for senses—and with such a criterion in place, there is no further need for the intuitive criterion. Some other criterion of identity would be needed, because without it, there would be nothing to distinguish between on the one hand the case where the speaker has an unclear grasp of sense *A*, and on the other hand has a clear grasp of a different sense *A'*. So this reformulation cannot be the whole story. It has been suggested before that Frege's intuitive criterion, or some version of it, should be restricted to cases where the subject fully understands the sense.<sup>10</sup> It would seem that

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<sup>8</sup> For the claim about their being non-temporal and non-spatial, see for instance Frege (1884, vii, 58, 61, 85, 93; 1918a, 25f; 1918b, 52). Their independent existence follows from their objectivity, which is stressed in for instance (1884, 26; 1918a, 69-70). For imperceptibility, see (1884, 85; 1924, 265). See also Burge (1992).

<sup>9</sup> Frege in fact makes this demand on occasion: "vorangesetzt wird, dass die Auffassung der Inhalte keine Schwierigkeiten macht", "provided that the grasp of contents presents no difficulties", Frege (1983, 212).

<sup>10</sup> Jeshion (2001) is one example of this kind of restriction—see pp. 965–966.

Frege never thought of such a restriction as the whole story about understanding and individuating senses—understanding is connected with the ability to *explicate* the concept. Presumably full understanding would be connected with the ability to fully explicate the concept. This still leaves us with a question concerning the identity criteria for senses: how are we to distinguish explicating one concept from explicating some other?

Another way to handle this worry would be to play down the Platonist understanding of what sense is like, and emphasize the way in which senses are to be seen as the way in which we think of references, thereby playing down Frege's view that senses are objects.<sup>11</sup>

There is one prominent passage in Frege, which might be taken to indicate that sense, unlike what is the case for some other abstract entities, perhaps is not completely independent of our activities and our opportunities to gain knowledge about sense:<sup>12</sup>

The being of a thought may also be taken to lie in the possibility of different thinkers' grasping the thought as one and the same thought. In that case the fact that a thought had no being would consist in several thinkers' each associating with the sentence a sense of his own; this sense would in that case be a content of his particular consciousness, so that there would be no *common* sense that could be grasped by several people. (Frege 1918b, 35)

This passage appears to support the claim that Frege occasionally thought that senses in some way could be partly dependent on human activities. I do not think that this is the correct interpretation of this passage. From the context it is clear that Frege has something else in mind. The situation described in the quoted passage is dismissed as spurious. The reason for this dismissal is that the transmission of thought would be endangered (and Frege repeatedly insists that we *do* transmit thoughts). The discussion of this particular problem ends with:

Our act of judgement can in no way alter the make-up of a thought. We can only acknowledge what is there. A true thought cannot be affected by our act of judgement. (Frege 1918b, 36)

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<sup>11</sup> For discussion of this, see Burge (1992) and Dummett (1991, ch. 12).

<sup>12</sup> Weiner takes it this way, in her (1995a) and (1995b).

It is true that Frege had a too stark dichotomy between on the one hand the completely subjective, and in his view incommunicable, and on the other hand completely objective, timeless or eternal, Platonistically conceived thoughts on the other hand. There was no place left for an intermediate category of the intersubjective, and several authors have tried to show that Frege's arguments for the eternal existence of thoughts are unsatisfactory, failing to establish his Platonism.<sup>13</sup> It may well be that some other position would have suited Frege's needs better or just as good, but there should be little doubt what the position was.

Frege conceived of our thinking as the grasping of senses: "In thinking, we do not produce thoughts, we grasp them." (1918a, 25) He also thought that it is only thanks to the Platonic nature of sense that it is possible for mankind to have a common stock of theories and knowledge. The premiss in his argument is that mankind has a common stock of knowledge, and that there are genuine cases of communication. Frege goes on to hold that the only explanation for this is that what we communicate could not be the sole property of a single speaker. What is communicated is something that is common to all (or at least to those engaged in communication). But it is not sufficient that what is communicated is common, in the sense of being intersubjective. It has to have a more permanent basis of existence. In fact it has to be thought of in Frege's preferred Platonist manner. Closely tied with this kind of Platonism is the view that it may often be quite difficult to have a clear grasp of the Platonistically conceived abstract objects. Frege held that we may have to struggle to arrive at a clear command of a concept. As a consequence of this, Frege thinks that we should not speak of the development of a concept, or of the history of a concept, but rather of the history of our attempts to grasp or articulate a concept clearly.<sup>14</sup>

Apart from the fact that it follows from the intuitive criterion that we are authoritative about sense, there is little *direct* evidence that Frege thought that we always, or regularly, are authoritative about sense. Some of Frege's views about our knowledge of abstract objects can be summarised as indicating that he thought we were authoritative about senses as well. One example of this might be Frege (1884, § 105), which holds that certain entities, being the "nearest kin" of reason, are utterly transparent to it. This

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<sup>13</sup> See Carruthers (1984a) and (1984b); Beaney (1996, chapter 7, especially pp. 213–224).

<sup>14</sup> Frege (1884, Introduction; 1891b, 134).

passage antedates the distinction between sense and reference, however, and may not be a blanket endorsement of the idea that senses are completely transparent to us. Frege could have had a restricted view of which abstract objects that qualify as the nearest kin of reason, and he could therefore in any case have wanted to exclude senses. Another passage which perhaps could be used to support the claim that sense must be completely known by a subject, is from Frege's letter to Jourdain in 1914. There he discards the idea that a Fregean thought—the sense of a sentence—could be (even in part) composed of physical objects: “It seems to be unreasonable that pieces of lava, *and also such that I have no knowledge of*, are parts of a thought”.<sup>15</sup> It is clear from the passage, and his other discussions of this issue, that he sees the main problem as lying in the supposition that actual physical objects make up a Fregean thought—this would be impossible to square with the eternal existence of thoughts—but here he locates part of the difficulty elsewhere: in thoughts being made up, even in part, by things I know nothing of. Admittedly, this may be reading too much into the passage, but it is noteworthy that Frege gives the argument he does, when his other views would have sufficed to settle this particular issue directly.

In addition to the claim that we may have difficulties arriving at a clear view of abstract objects like concepts, there are passages where Frege goes one step further, and holds that we sometimes do *not* have a clear grasp of senses. Here, Frege's idea is that we can grasp a sense, but that our command of this sense is blurred. The following passages are from “Logic in Mathematics”, written in 1914:

When we examine what actually goes on in our mind when we are doing intellectual work, we find that it is by no means always the case that a thought is present to our consciousness which is clear in all its parts. For example, when we use the word ‘integral’, are we always conscious of everything appertaining to its sense? I believe that this is only very seldom the case ... If we tried to call to mind everything appertaining to the sense of this word, we should make no headway. Our minds are simply not comprehensive enough. (Frege 1914, 209)

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<sup>15</sup> Emphasis added. German original, again with emphasis added: “Es scheint mir aber ungereimt, dass stücke Lava *und zwar auch solche, von denen ich keine Kenntnis habe*, Teile eines Gedanken sein sollen.” (1980b, 111)

How is it possible ... that it should be doubtful whether a simple sign has the same sense as a complex expression if we know not only the sense of the simple sign, but can recognize the sense of the complex one from the way it is put together? The fact is that if we really do have a clear grasp of the sense of the simple sign, then it cannot be doubtful whether it agrees with the sense of the complex expression. If this is open to question although we can clearly recognize the sense of the complex expression from the way it is put together, then the reason must lie in the fact that we do not have a clear grasp of the sense of the simple sign, but that its outlines are confused as if we say it through a mist. The effect of the logical analysis of which we spoke will then be precisely this—to articulate the sense clearly. (Frege 1914, 211)

Perhaps the sense appears to both [men] through such a haze that when they make to get hold of it, they miss it. One of them makes a grasp to the right perhaps and the other to the left, and although they mean to get hold of the same thing, they fail to do so. How thick the fog must be for this to be possible! (Frege 1914, 217)

Surely no arithmetical sentence can have a completely clear sense to someone who is in the dark about what a number is? This question is not an arithmetical one, nor a logical one, but a psychological one. We simply do not have the mental capacity to hold before our minds a very complex logical structure so that it is equally clear to us in every detail. For instance, what man, when he uses the word ‘integral’ in a proof, ever has clearly before him everything which appertains to the sense of this word! And yet we can still draw correct inferences, even though in doing so there is always a part of the sense in penumbra. (Frege 1914, 222)

In these passages Frege makes room for the possibility that a speaker can have some grasp of sense, but that this grasp is not sufficient for full authority.<sup>16</sup> In that case, the intuitive criterion will not work.

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<sup>16</sup> There are other passages. Frege (1906, 197) expresses a similar idea, although formulated in terms of “content”. Frege (1897, 138) explicitly says that our grasp of sense may be inadequate or blurred: “We might cite, as an instance of thoughts being subject to change, the fact that they are not always immediately clear. But what is called the clarity of a thought in our sense of this word is really a matter of how thoroughly it has been assimilated or grasped, and is not a property of thoughts.”

What could inadequate grasp of sense amount to? One way to explicate this is to talk about division of linguistic labour, and place the issue in the hands of the experts in the speaker's society (Putnam 1975). Then it would at least be possible to say about a single speaker in a society that he misunderstands a specific sense  $x$  (and not understands some other sense  $x'$  adequately), just in case there are experts in that society that do have an adequate grasp of sense  $x$ . This is, however, not what Frege is driving at. According to Frege, there may well exist several cases, where *no one*, not even the experts with the best theories, can be said to have a clear grasp of the sense of a certain expression. Frege uses examples such as 'number', where not even the best mathematicians have a clear grasp of the sense.<sup>17</sup> What would it in general mean to have the clear grasp of sense we are looking for? More on this below, in the concluding section, but the main idea is tolerably clear. If a speaker has a clear grasp of the sense of an expression—if the sense is clearly articulated—then the speaker knows full well what further things he is committed to, so that all the commitments involved in grasping that sense are laid bare. As in the passage from Frege (1897) which was just quoted (p. 138), it is a matter of the assimilation of the sense.

I will not make any further attempts to motivate these Fregean claims; it is sufficient that the claims about an inadequate grasp of sense are well entrenched in his theory of abstract objects, and that it shouldn't be seen as some kind of momentary and regrettable lapse.<sup>18</sup>

## 5. Handling the conflict

There is an obvious conflict here. If senses are individuated by the intuitive criterion, then the subject's considered judgments regarding possible combinations of attitudes are authoritative. And since Frege presents no other account of the identity or diversity of senses, we are left without

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<sup>17</sup> This is of course a central theme in Frege (1884), but is also used in Frege (1914), as on p. 242.

<sup>18</sup> Dummett refers to one of the just quoted passages from Frege (1914, 211), and comments: "It is to be hoped that this remark will not prompt anyone to try to reconstruct Frege's theory of the weather conditions in the space intervening between our minds and the realm of sense." (Dummett 1981b, 337) Frege had himself, however, used a fog or mist metaphor earlier in a similar context, see Frege (1884, vii-viii).

much of an account of what senses might be, at least if Frege's own strictures regarding the need for criteria of identity are to be accepted. If we stick with the only criterion of identity we are presented with—the intuitive criterion—the subject cannot have an incomplete grasp of senses. We stand without any alternative account of the identity and diversity of sense that works. This conflict is intimately connected with another issue as well. Among the cases where we are said to be ignorant of the senses, we find cases that were central to Frege's interests, such as *number*. In his earlier work, Frege had proceeded by definitionally equating numbers with extensions of concepts. This replaces a supposedly ill-understood notion with something that is precisely characterized. But if the sense isn't known, how could we then point to a definitional replacement?<sup>19</sup> In this section, I will consider various ways to handle the conflict, starting with suggestions that are of lesser interest, working my way towards those that may turn out to be more interesting.

The first suggestion is that Frege simply was confused: perhaps there really is an indissoluble conflict here, and Frege never realized it. Frege was not the kind of person to make mistakes of this kind. We should at least keep on looking for a more charitable interpretation of Frege.

Another idea is that Frege changed his mind, or that one of the two claims (the claim of the intuitive criterion and the claim about the unclear grasp of sense) doesn't really represent Frege's considered views on these issues. This is again possible, but there is no really good support for these suggestions. Both claims are well entrenched in Frege's thinking. The intuitive criterion can be found in the same paper as many of the passages about an incomplete grasp of sense (1914, 224ff). I think that none of the two claims can be discarded completely, although a solution of the conflict will probably have to put less weight on one of the claims.

The conflict could perhaps be avoided by stressing that there is a loophole: the intuitive criterion is about *differences* in sense, whereas the claim about incomplete grasp of sense is about knowledge of sense. There is then room for a position which holds that subjects really are authoritative about differences in sense, without always knowing what senses it is that they have authoritative knowledge about. They know whether this means the same as that, but they do not always know what it is that this or that

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<sup>19</sup> These issues are well discussed in Beaney's book, in Horty (1993) and (2007), and Tappenden (1995).

means. This is obviously not very attractive, and there is in fact no loophole available here. We could in that case simply construe senses as the equivalence classes of same-sense entities, in which case there would be no place left to talk about an inadequate grasp of senses—if a subject is authoritative about differences in senses, then he will turn out to be authoritative about grasp of senses as well.

Another suggestion might be that in cases where the sense of some expression is not known, then it is simply *impossible* to have rational thought. It seems that there are some hints at such a view in Frege: mathematics stands in need of firmer foundations than any that had been provided so far. But there are also opposing (rightly so, in my opinion) strands in his thinking. Mathematics is the crowning achievement of human thought, and if mathematical thinking were not rational, then what would be?

A further suggestion was mentioned earlier. The idea was that the intuitive criterion should be seen as hypothetical: if the subject has a clear grasp of the senses, then the intuitive criterion holds. One is almost inclined to say that this suggestion is *obviously* correct, and in a sense it is. But then, as I said above, two problems remain: what is it to grasp a sense clearly, and when *are* two senses identical? According to this suggestion, some other means of individuating senses takes precedence over the intuitive criterion, and the appeal to our intuitions concerning differences in cognitive value—the very intuitions that were used when Frege introduced senses in 1892—becomes problematic. Why should there be such a connection between sense and cognitive value? The answer that this is the function of sense for speakers with a clear grasp of sense is perhaps correct, but it is not very illuminating.

A different way of presenting this suggestion is that the claim about inadequate grasp of sense holds for actual speakers, whereas the intuitive criterion strictly applies only to ideal speakers. The passage from Frege quoted above (1914, 222) may substantiate this interpretation, since that passage describes inadequate grasp of sense as a matter of psychology, not logic or arithmetic. Given our constitution, we have to get at sense through language, featuring expressions we do not have complete command of, and the grasp of sense we get is at times distorted, hazy, or inadequate.

In another context, Frege held that there may exist beings that differ from ordinary human beings in being able to grasp sense directly, without the means of language. In fact, Frege has an even stronger point to make here: it is precisely *because* we human beings get at sense with the aid of

language—a human creation, created with a focus on other needs than displaying logical form—that we at times do not manage to get hold of sense properly:

There is no contradiction in supposing there to exist beings that can grasp the same thought as we do without needing to clad it in a form that can be perceived by the senses. But still, for us men there is this necessity. Language is a human creation; and so man had, it would appear, the capacity to shape it in conformity with the logical disposition alive in him. Certainly the logical disposition of man *was* at work in the formation of language but equally alongside this many other dispositions—such as the poetic disposition. And so language is not constructed from a logical blueprint. (Frege 1924/25a, 269)

The possible conflict is then avoided by holding that the perfect grasp of sense required for the intuitive criterion to work as a matter of psychological fact is something which we human language-users hardly ever attain, but that the intuitive criterion works for those case where the subject has some authoritative grasp of sense.

This is probably along the right lines, but then several things have to be done. One is that some other kind of criteria of identity and diversity for senses have to be derived, another is that we expect to be told when a speaker has the right sort of authoritative grasp of sense for the intuitive criterion to work. Also, if the intuitive criterion is applicable only for ideal, authoritative, speakers, then it is problematic to use sense as a central concept in a theory about normal speakers' understanding of language. We could argue that the concept of sense is little more than a harmless idealization of something we rarely manage to achieve. But in that case, the relation between on the one hand sense as understood in this idealized manner, and on the other hand sense as essentially connected with the use of language, becomes obscure.

There is a general tension between a linguistic and a non-linguistic conception of sense here. If grasp of sense is possible only through the linguistic expression of sense, then there is no problem seeing how we could fail to notice that two senses are the same: we could, as is at times the case with stipulative definitions, simply have cases where one expression is too long and complex to be handled with ease. Then it would be a *psychological* matter that our grasp of sense sometimes is inadequate. In the context of "Logic in Mathematics" (Frege 1914), Frege appears to have thought that

the value of stipulative definitions lay precisely in providing signs as suitable receptacles for sense, simply because our minds are not sufficiently capacious to handle the complexities we sometimes have to deal with:

If we tried to call to mind everything appertaining to the sense of this word, we should make no headway. Our minds are simply not comprehensive enough. We often need to use a sign with which we associate a very complex sense. Such a sign seems, so to speak, a receptacle for the sense, so that we can carry it with us, while being always aware that we can open this receptacle should we have need of what it contains. ... If therefore we need such signs—signs in which, as it were, we conceal a very complex sense as in a receptacle—we also need definitions so that we can cram this sense into the receptacle and also take it out again. So if from a logical point of view definitions are at bottom quite inessential, they are nevertheless of great importance for thinking as this actually takes place in human beings. (Frege 1914, 209)

This indicates Frege's abandoning the intuitive criterion for non-ideal speakers. The sense of one of the paired terms is too complex for the subject to have any attitudes about, and the whole issue of sameness of sense is simply bypassed. It is stipulated that the two expressions, the simple and the complex, have the same sense. And here we also have an implicit characterization of the plight of the non-ideal speaker—fettered by complexity considerations. Yet, as I said, Frege upheld the intuitive criterion even in this paper.

## 6. How we know senses

Frege held on to both the intuitive criterion and the view that we at times can—and in the interesting cases usually do—have an inadequate grasp of sense. This highlights a tension in his conception of sense. Senses cannot at the same time be the objects of propositional attitudes and be individuated by the subjects' attitudes towards these objects, unless we are dealing with ideal speakers. An ideal speaker is never side-tracked, bored, etc., and is in no way hindered by complexity. Hence Frege's demands on ideal speakers are very strong. Ideal speakers must have infinite minds. For finite minds, complexity will always have to be factored in. Ideal speakers are correct about the objects of their propositional attitudes, and if they

hold different attitudes towards two senses, these senses are not identical. In general, we don't arrive at that state (though there may be some cases where we have that kind of knowledge). Far from being anomalies, the possibility of an inadequate grasp of sense is in fact of central importance for Frege's philosophical project.

The project of giving a firm foundation for arithmetic forced Frege to face the issue how his proposed reconstruction of arithmetic was connected with the starting point, arithmetic as it was known and practiced. It was natural for Frege to conceive of the knowledge imparted by his reconstructions as a gradual unfolding of a sense already given, but imperfectly understood. The task of the proposed reconstruction was to better articulate and understand the notions involved. But Frege never succumbed to some kind of Cartesian scepticism concerning mathematical knowledge. He had the working mathematician's faith that they were at least getting *something* right, even if the various paradoxes and conceptual revolutions of nineteenth century mathematics showed that there was foundational work to be done. This attitude can be seen in a letter to Hilbert from 1895, where Frege discusses the merits of, and need for, symbolization:

A purely mechanical formalization is dangerous 1 for the truth of the results, 2 for the fruitfulness of the science. It appears that the first danger can be dealt with completely by the logical perfection of the symbolization. As for the second, the science would be brought to a standstill if the formulaic mechanism would take over to such an extent that it suffocated the thought completely. I would still not like to think of such a mechanism as completely useless or harmful. On the contrary, I think it is necessary.<sup>20</sup>

From this passage it is clear that Frege thinks that it is fundamentally possible to have some degree of access to the objects of mathematics before we start formalization. Formalization is perhaps necessary for full understand-

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<sup>20</sup> Frege, letter to Hilbert, Oct 1, 1895, Frege 1980b, 4f. Translation by the author. German original: "Ein bloss mechanisches formeln ist gefährlich 1. für die Wahrheit der Ergebnisse, 2. für die Fruchtbarkeit der Wissenschaft. Die erste Gefahr lässt sich wohl fast ganz durch die logische Vervollkommnung der Bezeichnung beseitigen. Was die zweite betrifft, so würde die Wissenschaft zum Stillstande gebracht, wenn der Formelmechanismus so überhand nähme, dass er den Gedanken ganz erstickte. Dennoch möchte ich solchen Mechanismus keineswegs als ganz unnütz oder schädlich ansehen. Im Gegenteil glaube ich, dass er notwendig ist."

ing and sharp explication of the concepts, but ordinary practices do not leave mathematicians in the dark.

Frege's view on our knowledge of abstract entities was not tied with the usual Platonist emphasis on intuition, or using *seeing* as the guiding metaphor. Frege basically thought of such knowledge in more practical terms, as a kind of *doing*.<sup>21</sup> And in handling, as it were, the senses, it is possible to step by step arrive at ever clearer articulations of the sense—giving up the ideal of such access to abstract entities as a kind of glorified seeing tends to remove the temptation to think of such access as infallible. The temptation to think of such access as giving us direct insight into the nature of these entities is also reduced. The passage from Frege 1897, already referred to, is both an endorsement of the view that senses can be inadequately grasped and that it is not inherent in the nature of senses that they are transparent:

what is called the clarity of a thought in our sense of this word is really a matter of how thoroughly it has been assimilated or grasped, and not a property of a thought. (Frege 1897, 138)

To think of senses as entities that normally, or even invariably, obey the ideal of transparency of mental content that is inherent in the intuitive criterion reverts to a recognizably Cartesian conception of the inner, and makes Frege's views harder to understand. This does not prevent the intuitive criterion from being useful as a kind of ideal at the end of inquiry.

The connection between the problem at hand and the position I called "rationalist pragmatism" in the first section is pretty straightforward, now that we have come this far. Making the grasp of sense a matter of what we can *do* with the sense relieves much of the tension that should have troubled Frege, and it also introduces views and themes that it would not be unfair to call "pragmatist". A more comprehensive treatment of this strand in Frege's thought lies well beyond the scope of this paper, so here I will just give a very brief sketch of what I mean by this, and offer some textual backing for my claims.

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<sup>21</sup> The differences between the traditional, vision-based, conceptions of knowledge of abstract entities and Frege's more pragmatic views are real enough, but there are elements of a vision-based conception in Frege as well. Since Frege repeatedly used the fog and mist metaphors to characterize our inadequate grasp of senses, he must have found it natural to think of the process in terms of visual metaphors.

The tensions in Frege's conception of sense display one central strand in his views concerning the point of his philosophical enterprise. A part of this enterprise was devoted to the logicist program of providing firm, logical, foundations for mathematics. This meant that he had to show that certain entities deep down were something they on the face of it were not, as when he held that numbers were the extensions of concepts. A proposed *analysans* would have to have at least something in common with its *analysandum*, or else the point of the project was left uncertain. To leave the reference intact would be insufficient, and to leave the sense intact would lead us directly to the paradox of analysis, if the intuitive criterion were upheld: how could the proposed analysis be of any value, if all it did was to conserve the sense? Frege's thought was that, when doing the kind of foundational work he was engaged in, we present a reconstruction of the problematic notions, rather than an outright sense-preserving analysis. An ill understood notion is replaced by a streamlined and better articulated notion. This is quite close to Carnap's ideas concerning explication and Quine's views on regimentation. But if the success of the proposed reconstruction is not to be judged solely by the intuitive criterion, how is it to be judged? Here Frege's answers are somewhat sketchy, but in the early period, he tended to stress *fruitfulness* as the central criterion:

All these concepts have been developed in science and have proved their fruitfulness. For this reason what we may discover in them has a far higher claim on our attention than anything that our everyday trains of thought might offer. For fruitfulness is the acid test of logic, and scientific workshops the true field of study for logic. (Frege 1880/81, 33)

In the early period, Frege's demand is that definitions should be fruitful (1884, 100f), and much of his early thought, at least, displays a general outlook that has some clearly pragmatist features. Peirce took the "Principle of Pragmatism" to be the following principle about meaning:

In order to ascertain the meaning of an intellectual conception we should consider what practical consequences might conceivably result by necessity from the truth of that conception; and the sum of these consequences will constitute the entire meaning of the conception. (Peirce, quoted in Audi 1996, 566)

Guiding in the pragmatist conception of meaning is the idea that differences in meaning always must entail possible differences in experience. Prag-

matism is thus historically usually allied with some form of empiricism. In Frege, however, we find a kind of rationalist version of this idea. For logic, Frege says the following:

One must always hold fast to the fact that a difference is only logically significant if it has an effect on possible inferences. (Frege 1880/81, 33fn)

Taken in isolation, this quotation is admittedly rather weak as evidence for a pragmatist view, since it is hard to understand what other grounds there might be for logical differences, apart from differences in inferences. Frege uses this view in arguing against Wundt's representation of logical inferences. What is more important for the present issue is that Frege's views on the basic nature of our knowledge of senses, and on a priori thought in general, diverge from the standard Platonist picture of knowledge as a kind of seeing. The pragmatist idea is that our knowledge of meaning is not so much a matter of *seeing* things in the abstract realm—it is better seen as a question of *doing*, of making the right transitions, of connecting this with that, of assimilating a concept into a network of conceptual connections.<sup>22</sup> And it is not surprising if it should turn out that this command is not something which the single speaker could display all at once, not even to himself.<sup>23</sup> For Frege, knowledge was always a matter of *doing*, which his choice to talk of *grasping* senses indicates. Knowing something about abstract objects is not tied to intuiting them, or seeing them in a third realm—it is tied to knowing what to do with them. A typical early example is the following:

[W]hat is common to lengths and surfaces, escapes our intuition. This comes out most clearly in the case of an angle. No beginner will get a correct idea of an angle if the figure is merely placed before his eyes. this is what has occasioned numerous attempts to give an explanation of an angle, even though the situation is at bottom exactly the same in the case of a length, except that this idea is more familiar to us from our

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<sup>22</sup> Frege's consistent rejection of appeals to intuition, and his insistence that we ground knowledge of something by providing proofs, is in line with this general view.

<sup>23</sup> Brandom discusses this feature of Frege's thought, and dubs it "tactile Fregeanism", in Brandom (1994, 579ff), and in (2008, 51–52).

ordinary lives. *If a beginner is shown how to add angles, then he knows what they are.* (Frege 1874, 56, emphasis added)

If this is Frege's most basic conception of what knowledge of the third realm amounts to, we should not be surprised to find that our knowledge can be improved, that there is no guarantee that it is authoritative or immediate.<sup>24</sup> It would not be too wide of the mark to call this kind of view *rationalist pragmatism*. It also clearly has parallels with Brandom's inferentialism.<sup>25</sup> What we *do* with stuff is more important than what it represents. Knowledge-how is more basic than knowledge-that in this area.

Frege's views here have some central aspects in common with classical pragmatism, but steer clear of the empiricism with which pragmatism is usually associated. It sees our current position as a state of knowledge, but knowledge that can and should be improved, so a wholesale scepticism is beside the point. This is a kind of position which has echoes in Peirce and in Neurath's celebrated boat metaphor.<sup>26</sup> Knowledge can be improved and sharpened, and put on a firmer basis. This firmer basis is provided by the new tools that the *Begriffsschrift* can provide for the mathematician, and the kind of full knowledge of abstract objects that is the mark of full understanding is probably attainable *only* by use of the symbolic tools of the *Begriffsschrift*. Knowledge in this area is a kind of knowing *how*: knowing how to add angles, how to explicate concepts fully, how to draw connections in the web of concepts making up the abstract world of mathematics, a world which is independent of us, but possible to arrive at knowledge about. So even if pragmatism is a sort of hazy and much abused term, I think the term fits many of Frege's deeply held beliefs about our knowledge of abstract objects. This position is still little explored, but merits further study.<sup>27</sup>

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<sup>24</sup> Belabouring Frege's tactile metaphors, one could say that there is at times *groping* involved in trying to *grasp* a sense.

<sup>25</sup> See Brandom (1994), (2000) and (2008).

<sup>26</sup> In Frege (1924/25b), he states: "I, for my part, never had any doubt that numerals must designate something in arithmetic, if such a discipline exists at all, and that it does is surely hard to deny." Frege is not doubting arithmetic and the existence of numbers from the bottom up; he is trying to find a clearer and more perspicuous foundation for something.

<sup>27</sup> Burge (2005) has done much to investigate this kind of position

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